



POST-QUANTUM CRYPTOGRAPHY

By

Aibanjali Venkatesh

INTRODUCTION

WE UNDERSTAND THE POTENTIAL OF QUANTUM COMPUTERS TO THREATEN THE SECURITY OF CURRENT ENCRYPTION SCHEMES.

THE REACTION TO THIS RISK IS THAT MUCH EFFORT HAS BEEN PUT INTO CLASSICAL (NON-QUANTUM) ALGORITHMS THAT CAN RESIST QUANTUM ATTACKS.

THE ILLUSTRATED GUIDE TO POST QUANTUM CRYPTOGRAPHY DESCRIBES THE NATURE OF THESE CLASSICAL ALGORITHMS.

STARTING FROM WHY THIS NEEDS TO BE ADDRESSED, WE GO ON TO EXPLORING A GLOBAL COMPETITION TO IDENTIFY SUITABLE ALGORITHMS.

FAR TOO MANY BRILLIANT MINDS HAVE CONTRIBUTED TO THE RESEARCH TO MENTION INDIVIDUALLY.

ONCE WE'VE COVERED THE WORKINGS OF A HANDFUL OF ALGORITHMS, WE ALSO TAKE A LOOK AT WHAT IT MEANS TO MIGRATE TO A POST QUANTUM CRYPTO SCHEME.

PRE READ THESE ILLUSTRATED GUIDES FROM THOUGHTWORKS

HOW TO TELL SECRETS

THE STORY OF QUANTUM COMPUTING

GUIDE TO AES

WEB 3 — THE PART ON MERKLE TREES

WHAT IS PQC?

POST QUANTUM CRYPTOGRAPHY OR PQC IS THE FOCUS ON DEVELOPING CLASSICAL ALGORITHMS THAT ARE SAFE FROM ANY DECRYPTION ATTEMPTS BY QUANTUM ALGORITHMS

IS PUBLIC KEY CRYPTOGRAPHY BROKEN?

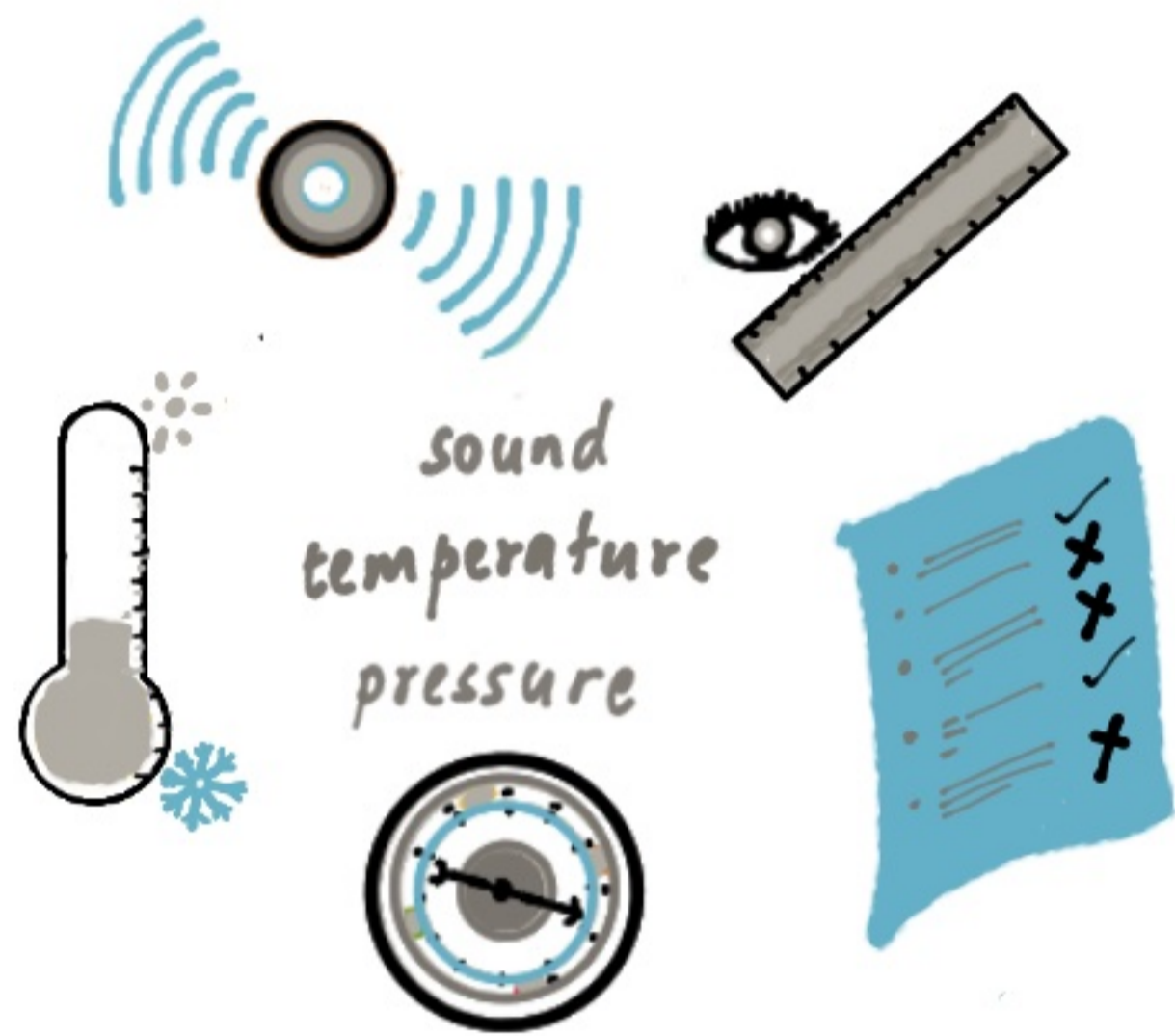
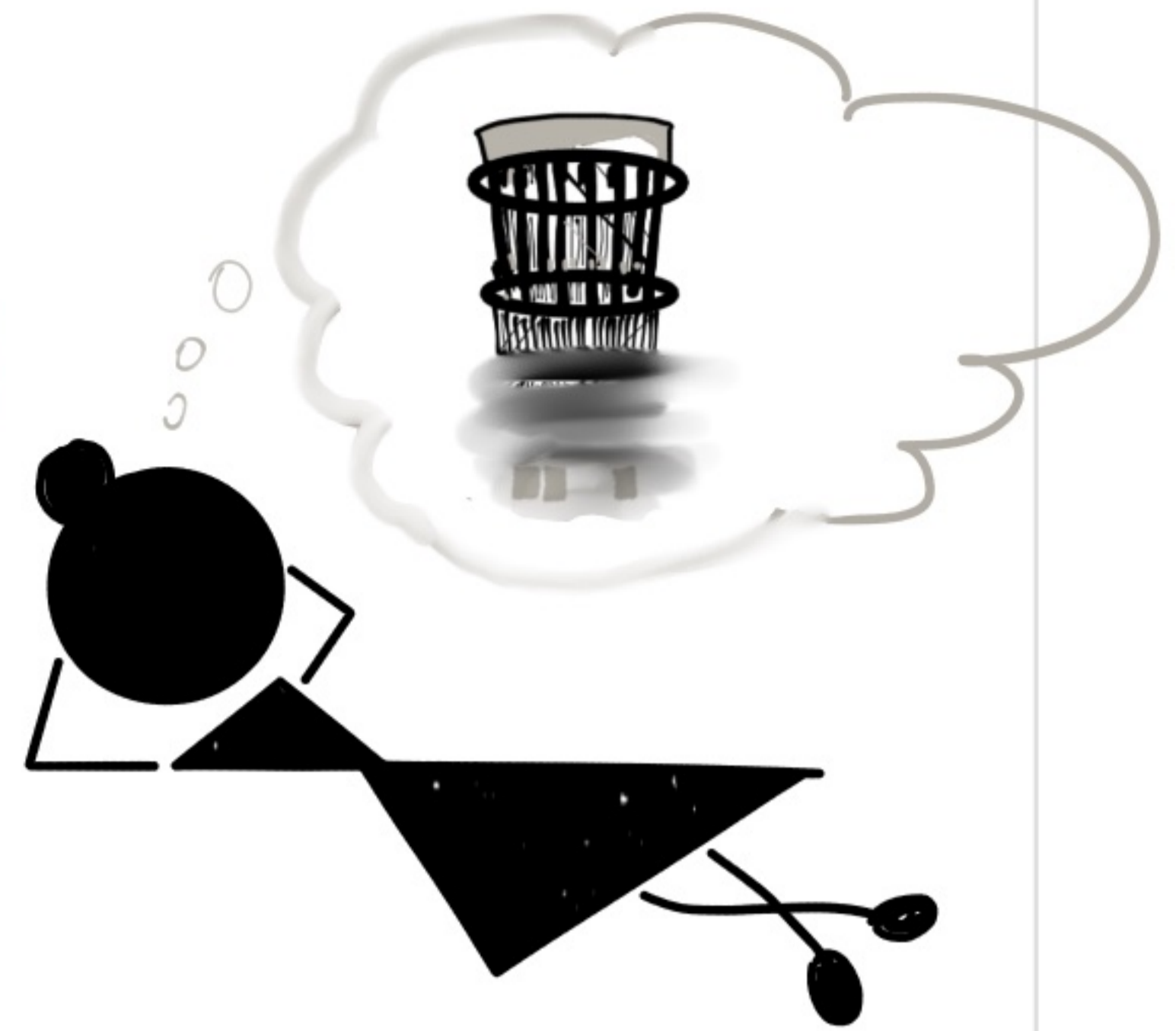
NO!



THE PUBLIC KEY CRYPTOGRAPHY IN USE
WORKS ONLY TO DEFEND AGAINST CLASSICAL
- NOT QUANTUM COMPUTERS

WHY NOW?

WHY THE SEARCH FOR
POST-QUANTUM CRYPTOGRAPHY?
A USEFUL QUANTUM COMPUTER
ISN'T HERE ... YET!



A QUANTUM COMPUTER
IS STILL SUSCEPTIBLE TO
NOISE AND ERRORS

RSA, ECC & OTHER
ASYMMETRIC ENCRYPTION &
DIGITAL SIGNATURE METHODS
ARE STILL STRONG - RIGHT?



SYMMETRIC KEY

4F	20	1E	01	3E	63	47
77	66		75	6E	2D	
12	06		68	20	12	
2C	22		2E	41	11	

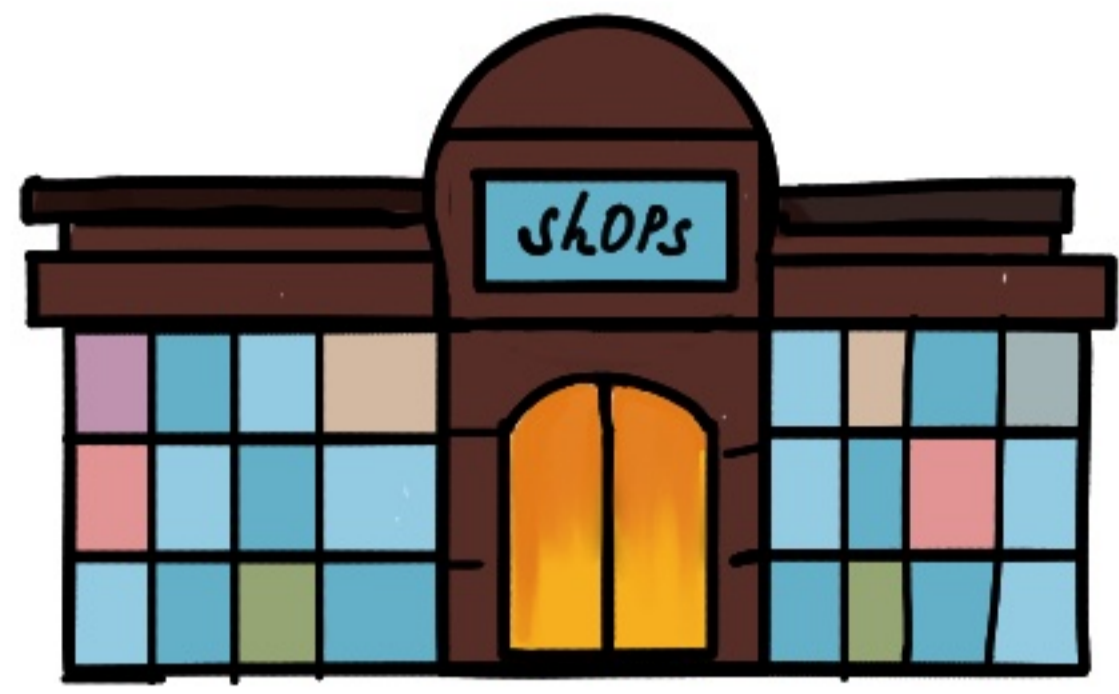
256

00	4	1	77
0E	9F		31
0A	16		8D
39	61	5A	43

AES

BESIDES, AES 256 IS
QUANTUM-PROOF!

REASON 1: ATTACKS



BUSINESSES/BANKS

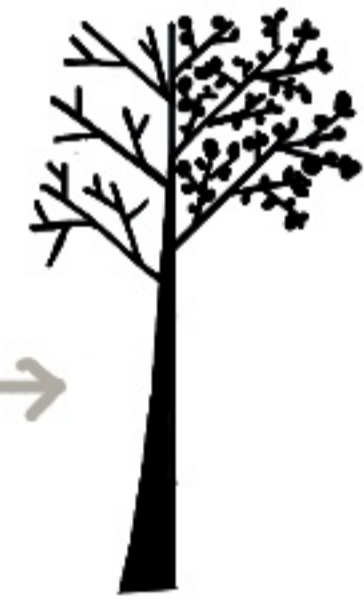


GOVERNMENTS

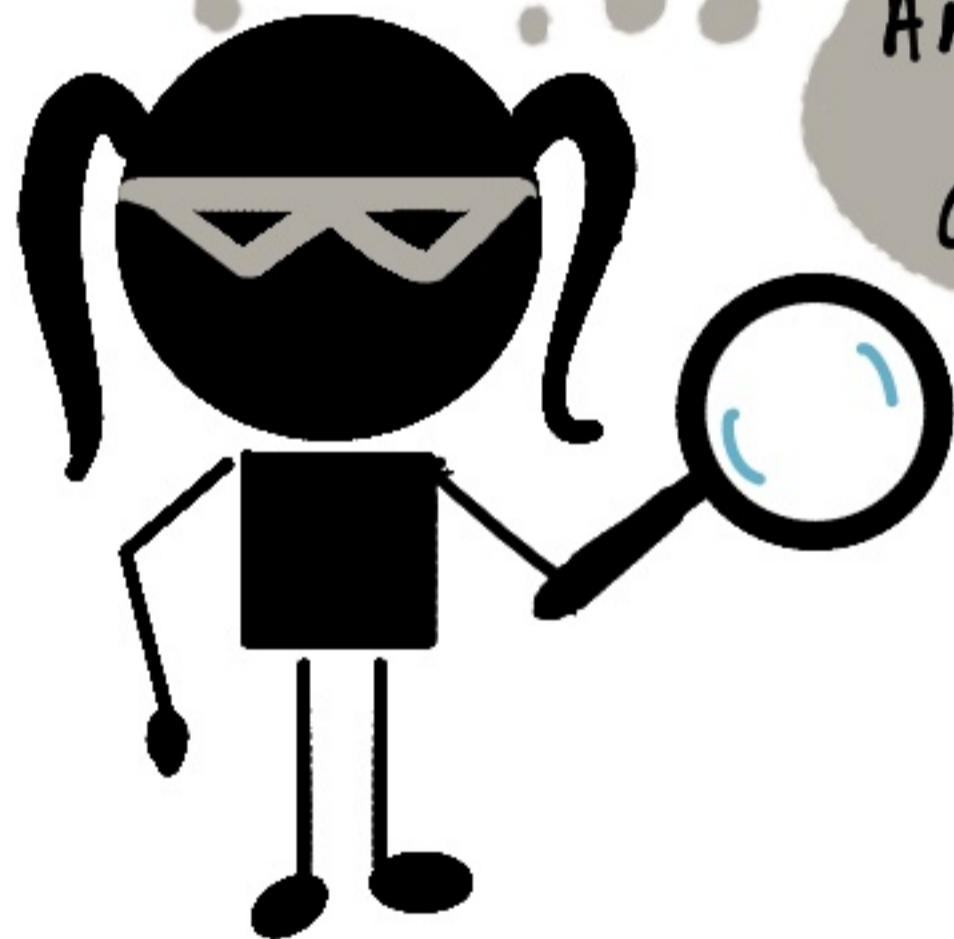
STORE DATA ABOUT CUSTOMERS/CITIZENS



USUALLY ENCRYPTED WITH RSA/ECC



'Y2Q' will be here....!



And I could carry out

Y2Q } YEARS TO QUANTUM-speculative

a) HARVESTING ATTACK

- DECRYPT YOUR ENCRYPTED DATA

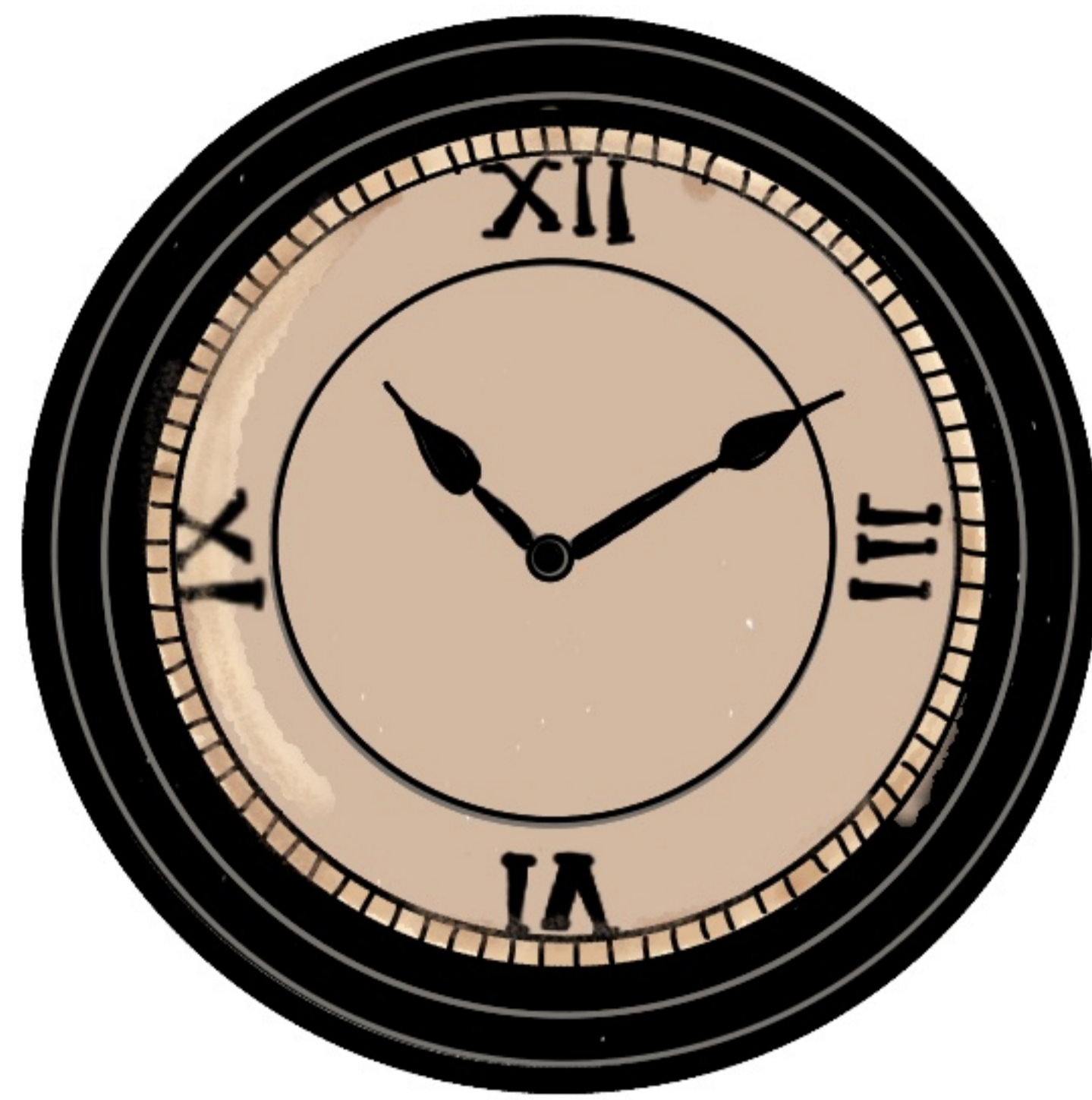
b) REWRITE HISTORY -

BY FAKING A DIGITAL SIGNATURE ON PAST RECORDS

USING THAT QUANTUM COMPUTER

P.S. I might be a lot greyer!

REASON 2: TIME



ACCORDING TO THE NIST, IT TAKES A LONG TIME
TO ROLL OUT NEW ENCRYPTION AT SCALE

MODERN PUBLIC KEY CRYPTOGRAPHY INFRASTRUCTURE HAS TAKEN

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----

20 YEARS TO DEPLOY

SO NOW MIGHT BE A GOOD TIME TO START BEING CURIOUS
ABOUT HOW TO PREPARE IT SYSTEMS FOR THE FUTURE

MOSCA'S THEOREM

HOW LONG DOES DATA
NEED TO BE SECURE?

x

HOW LONG UNTIL A
QUANTUM SAFE SOLUTION?

y

HOW LONG UNTIL A USEFUL AND
POWERFUL QUANTUM COMPUTER?

z

IF

x

+

y

>

z

WE HAVE A PROBLEM

Highlights the
store now
decrypt later
problem



MICHELE MOSCA

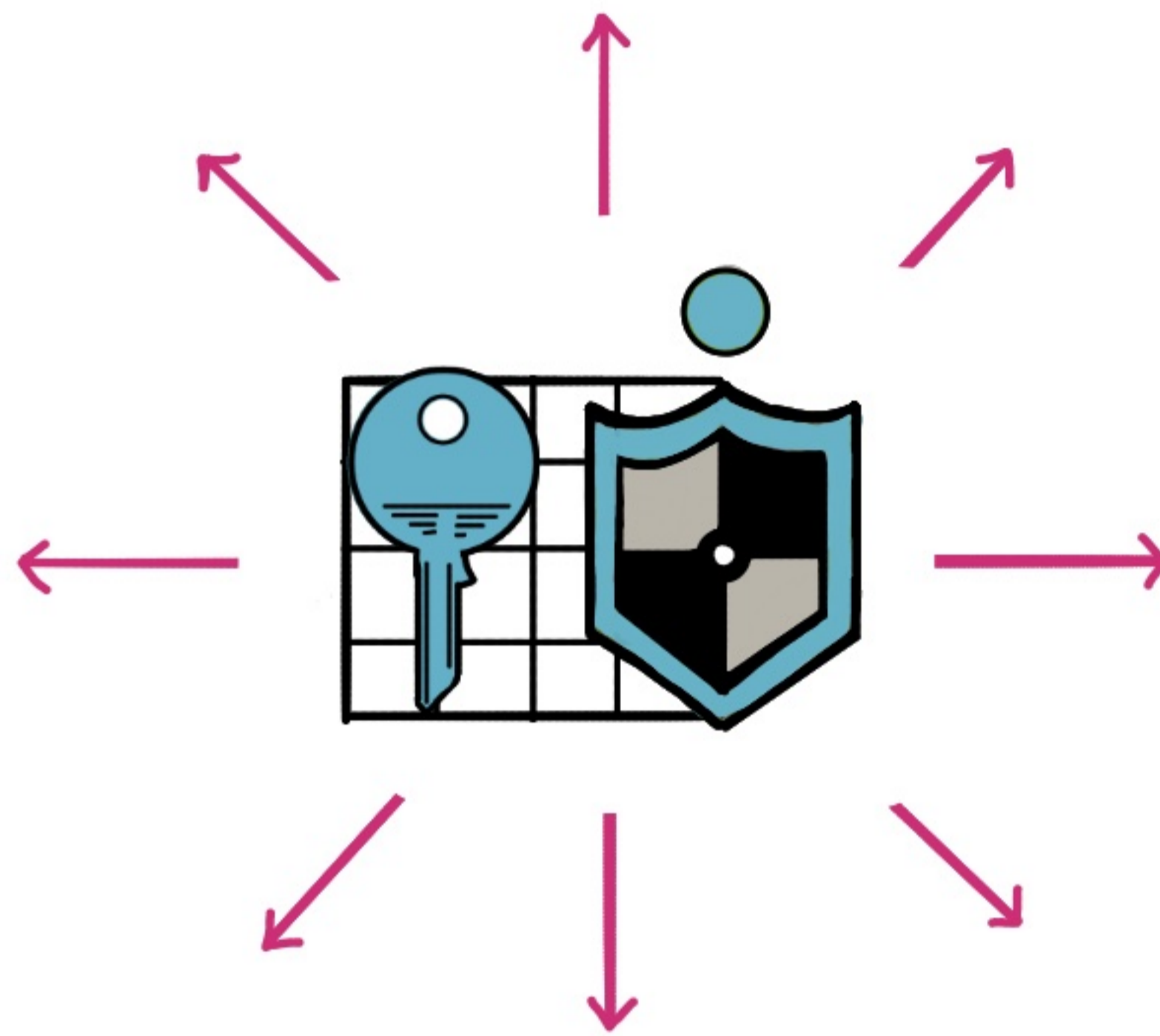
MATHEMATICIAN &
COMPUTER SCIENTIST

WHAT OPTIONS DO WE HAVE?

OPTION - 1

USE AES, HOWEVER...

AES IS QUANTUM-SAFE
SYSTEMS AROUND IT ARE NOT



WHAT'S THE SOLUTION?

INCREASE THE SIZES
OF AES KEYS

+

FIND SECURE WAYS
TO DISTRIBUTE THE KEY

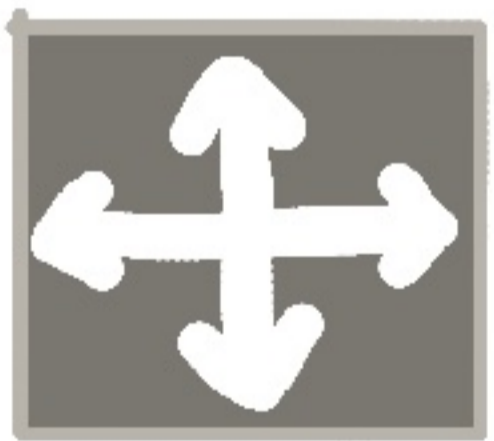
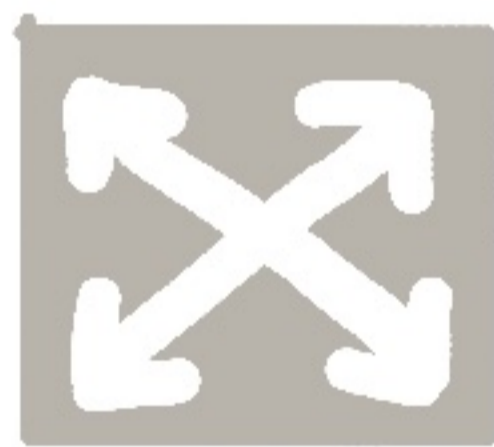
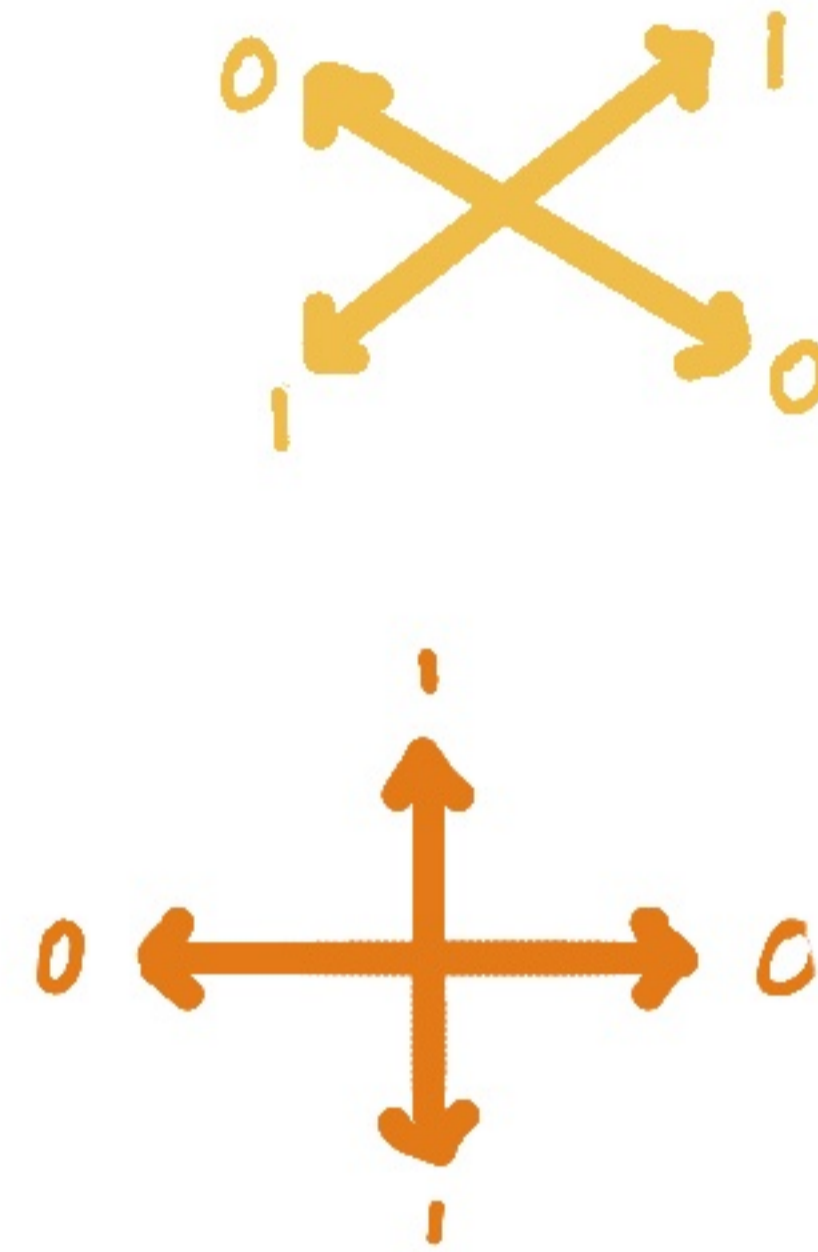
WHAT OPTIONS DO WE HAVE?

OPTION - 2

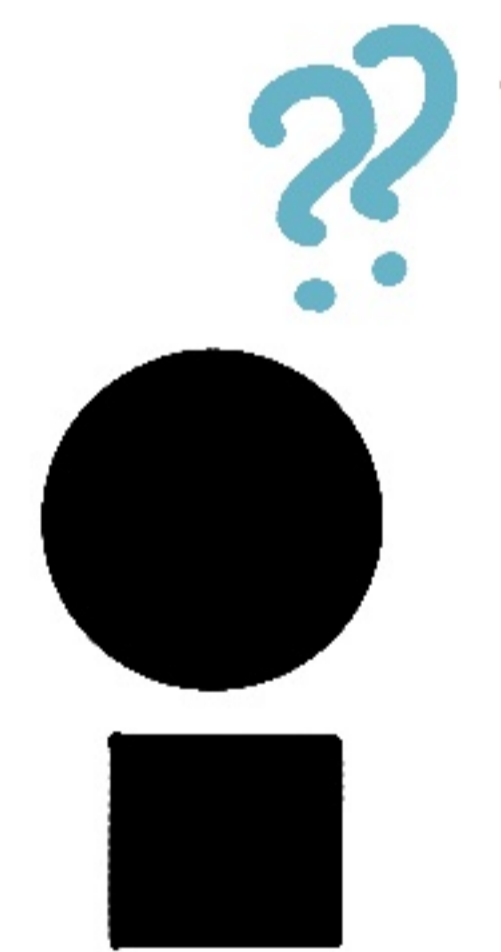
USE QUANTUM KEY DISTRIBUTION - QKD




ALICE
SENDS
POLARISED LIGHT
WHICH ENCODES BITS
RANDOMLY
IN ONE
OF 2
METHODS



BOB
MUST GUESS
WHICH METHOD
AND
'READ'
THE PHOTON



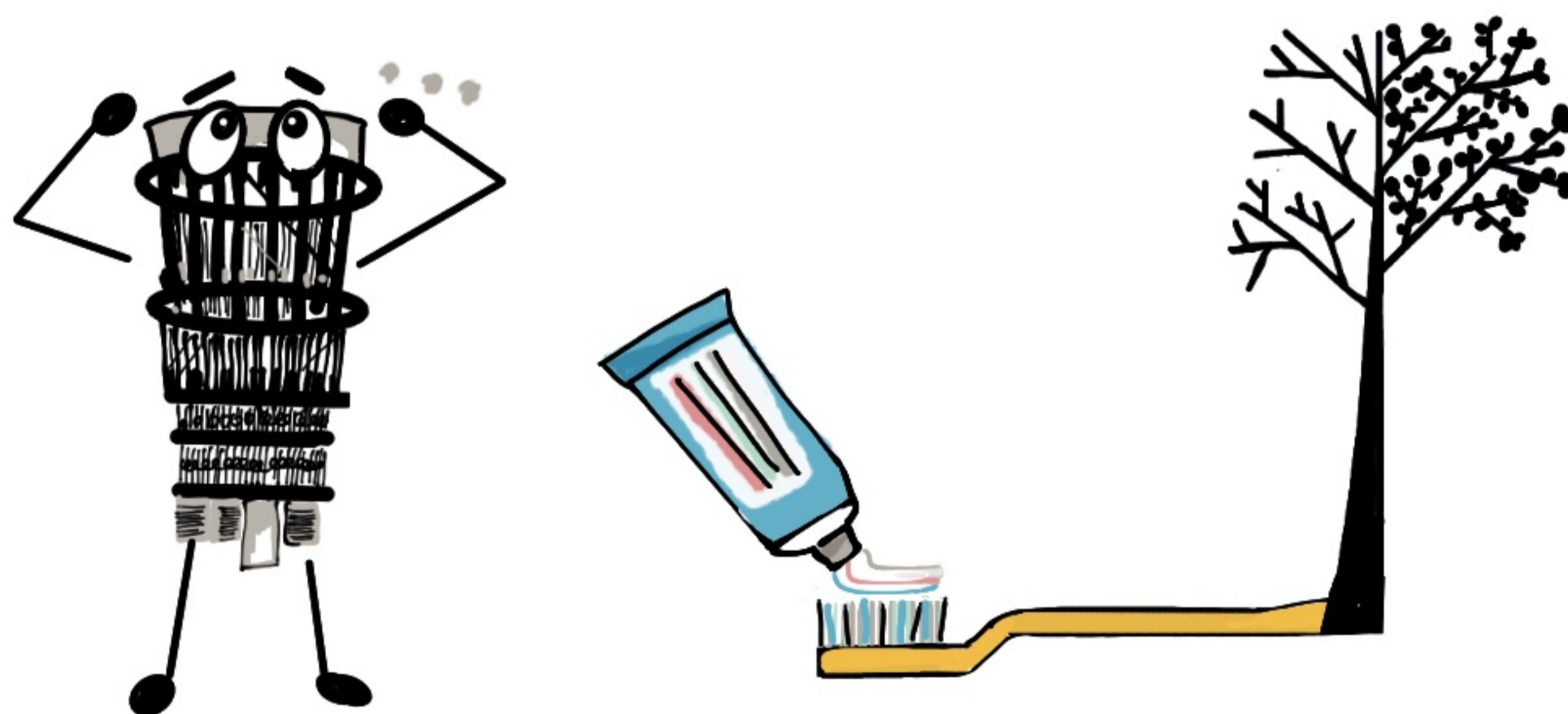
 THIS IS NOT YET
FEASIBLE AT SCALE

WHAT OPTIONS DO WE HAVE?

OPTION - 3

FIND 'QUANTUM-SAFE' ENCRYPTION

ALGORITHMS HARD FOR
QUANTUM COMPUTERS TO CRACK



CONSTRUCT COMPLEX
TRAPDOOR FUNCTIONS

EASY TO DO
HARD TO UNDO

CLASSICAL +
FOR QUANTUM
COMPUTERS

THIS BOOK IS ABOUT QUANTUM-SAFE ALGORITHMS
AND HOW THEY WORK

TO CLARIFY

QUANTUM ALGORITHMS

≠

QUANTUM RESISTANT ALGORITHMS

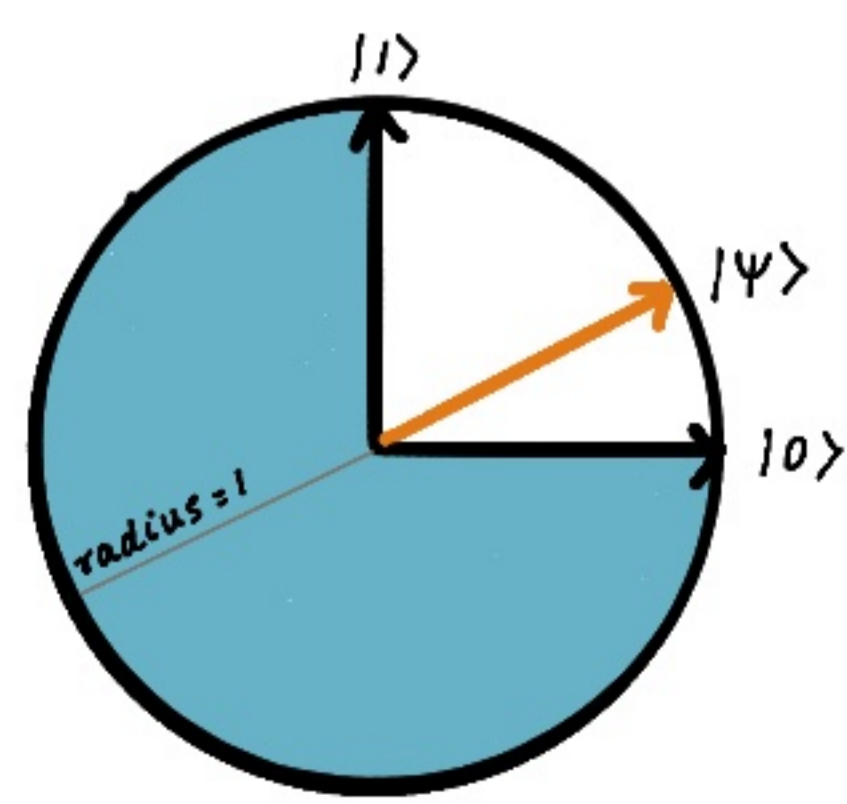
- SHOR'S ALGORITHM
- GROVER'S ALGORITHM

- KYBER
- SPHINCS+

QUANTUM ALGORITHMS
USED BY

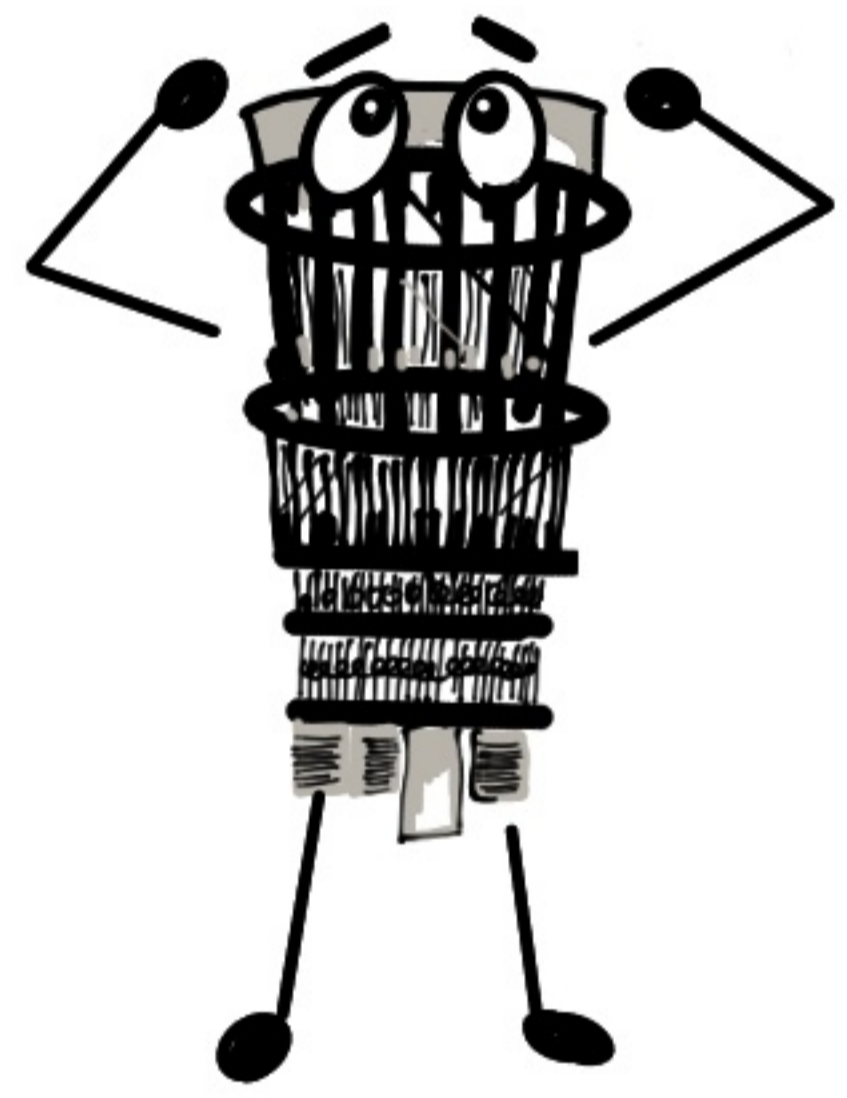


QUANTUM COMPUTERS
TO SOLVE PROBLEMS

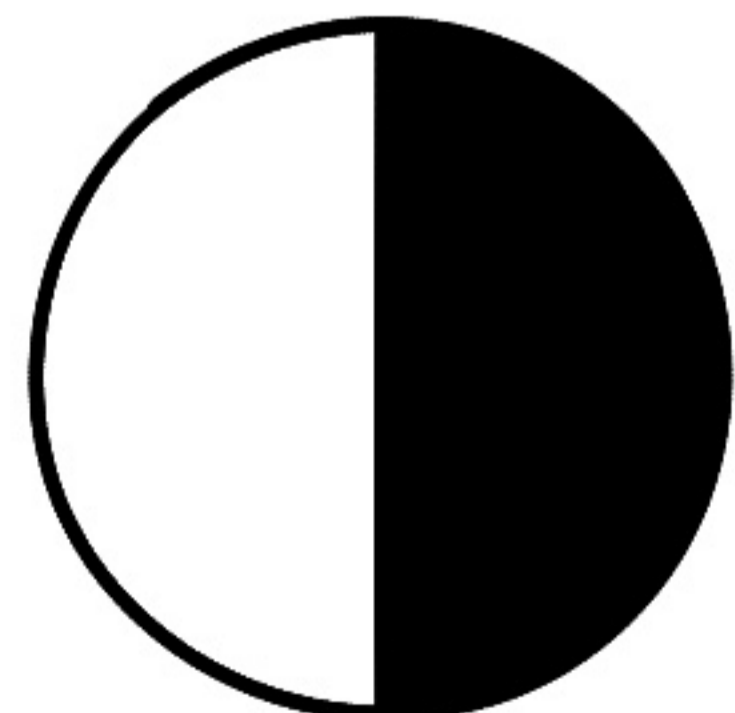


✓ YES, QUBITS ARE USED

CLASSICAL ALGORITHMS
THAT



QUANTUM COMPUTERS
CANNOT CRACK/SOLVE



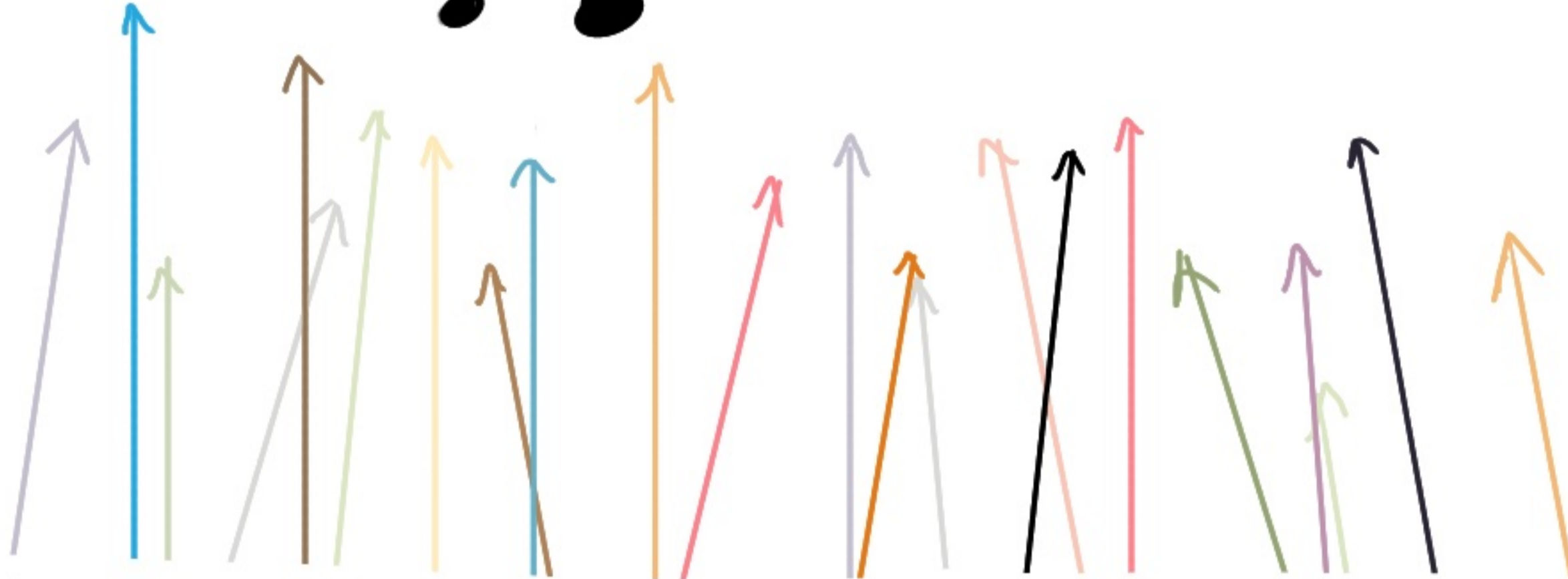
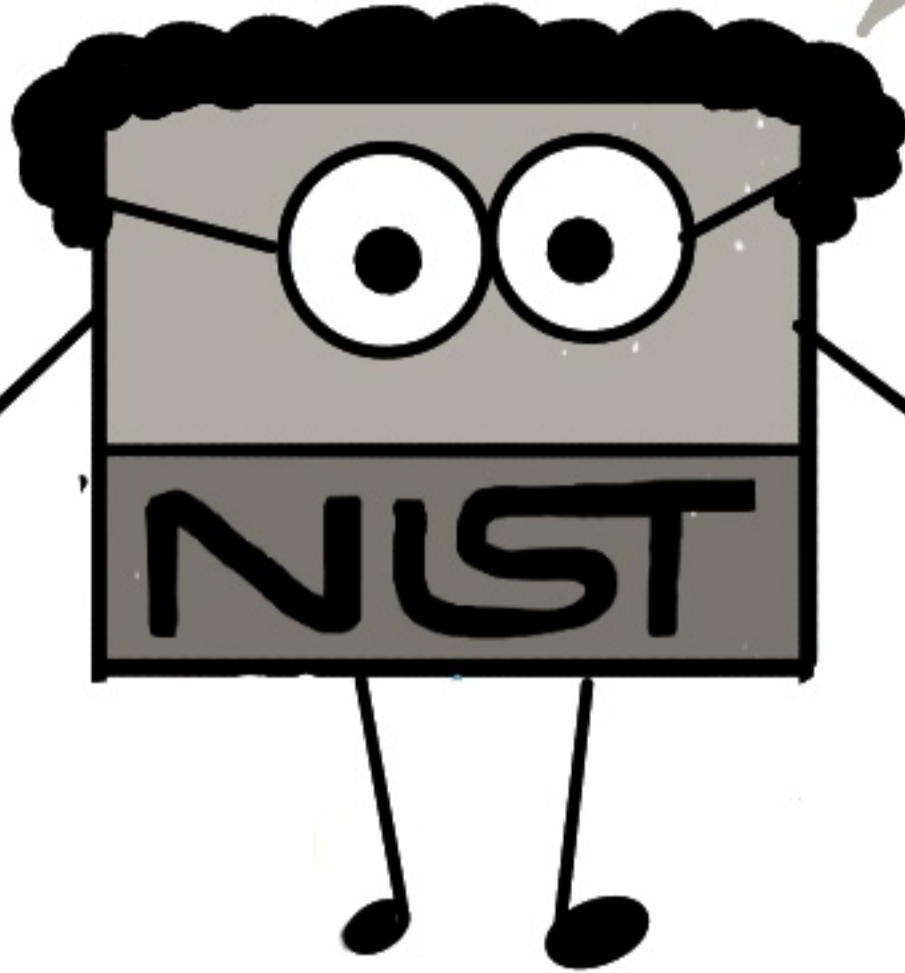
✓ GOOD OLD BITS ARE USED

IT BEGAN IN 2016

NIST- NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY



WE CALL FOR IDEAS FOR PQC ENCRYPTION & SIGNATURE SCHEMES



69 VIABLE CANDIDATES SUBMITTED AND DISCUSSED IN ^{public} GOOGLE GROUP PQC-FORUM



SELECTION

CRITERIA

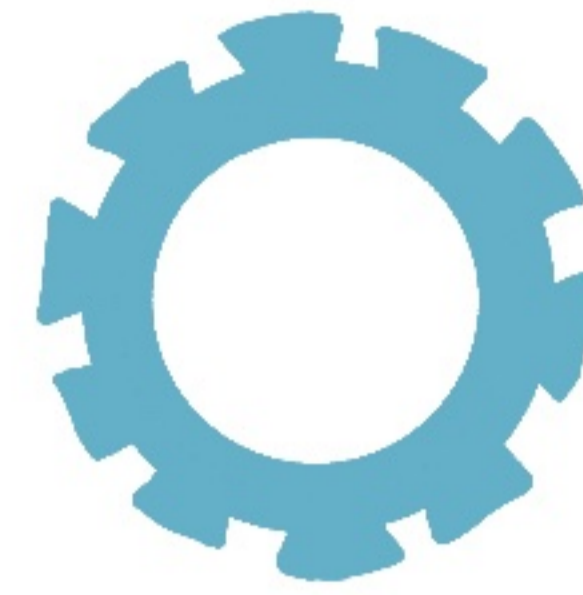
SECURE



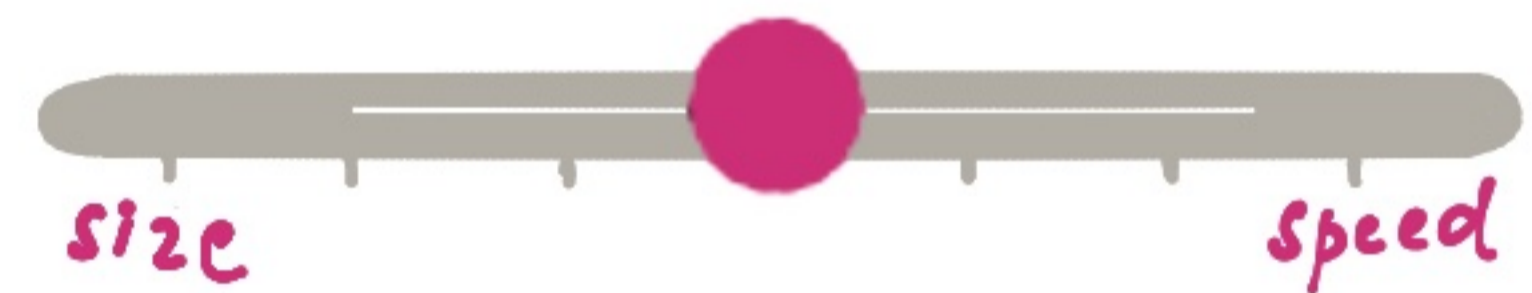
- AGAINST ATTACKS FROM BOTH CLASSICAL AND QUANTUM COMPUTERS
- BASED ON A HARD PROBLEM

CONFIGURABLE

- 5 LEVELS OF SECURITY



- PICK THE TRADEOFFS



EFFICIENCY



- KEY SIZES
- SIGNATURE SIZES
- CIPHER TEXT SIZES
- MEMORY
- BANDWIDTH

OTHERS

SIMPLICITY

EASE OF ANALYSIS

RESILIENCE TO

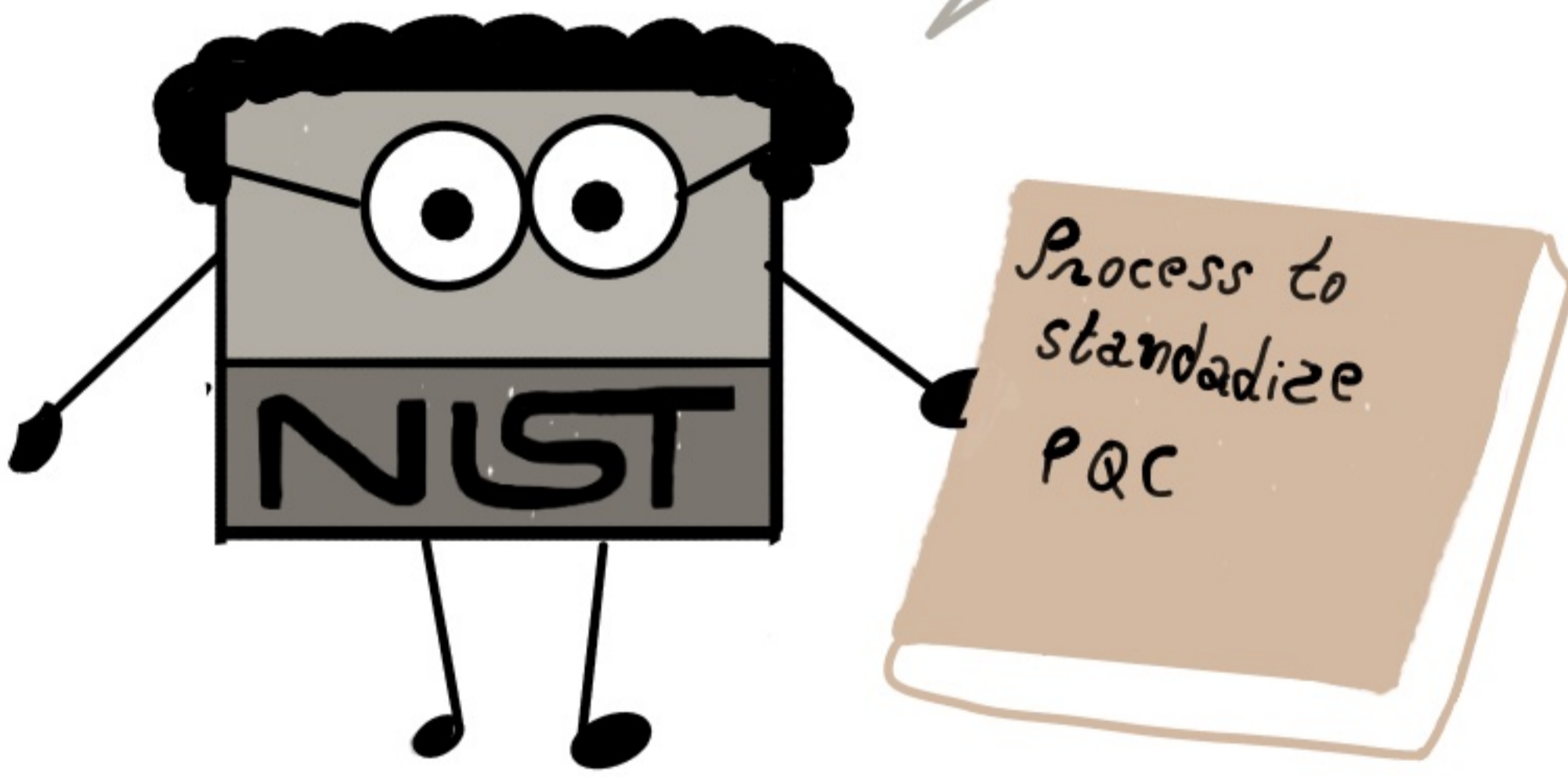
SIDE-CHANNEL ATTACKS

COMPATIBILITY WITH

EXISTING PROTOCOLS

IN 2022

4 ALGORITHMS THAT ARE DESIGNED TO WITHSTAND THE ASSAULT OF A FUTURE QUANTUM COMPUTER AND WILL BE PART OF THE STANDARD....



GENERAL ENCRYPTION



CRYSTALS-KYBER

DIGITAL SIGNATURES



CRYSTALS-DILITHIUM



SPHINCS+

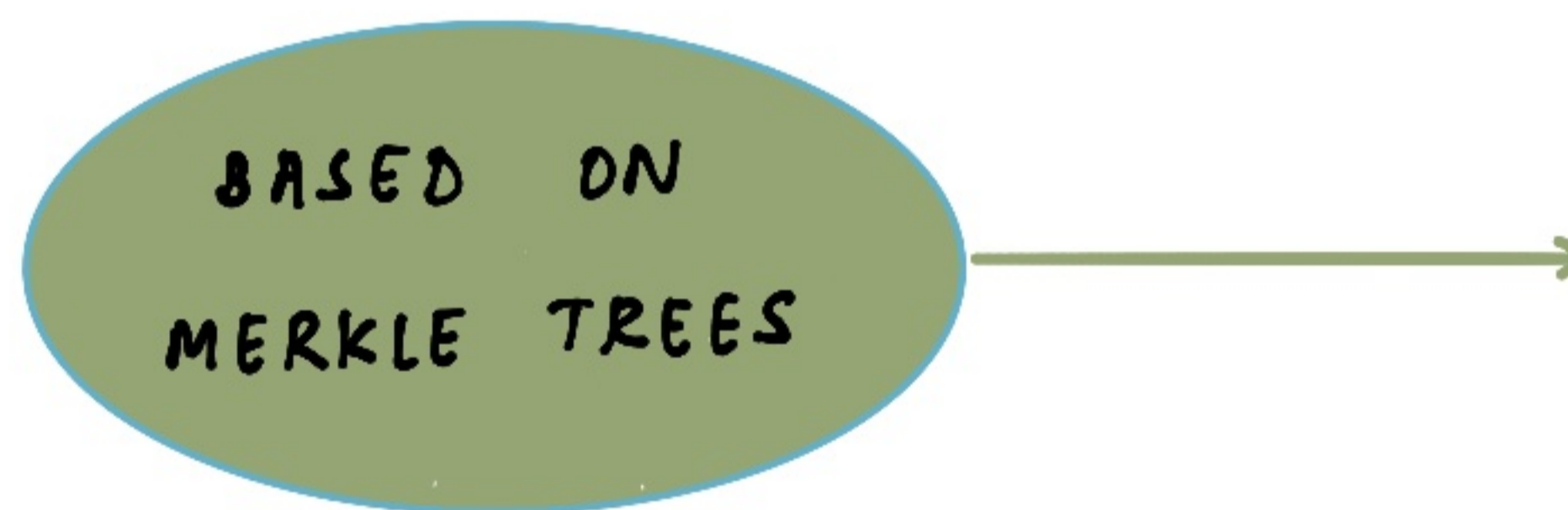
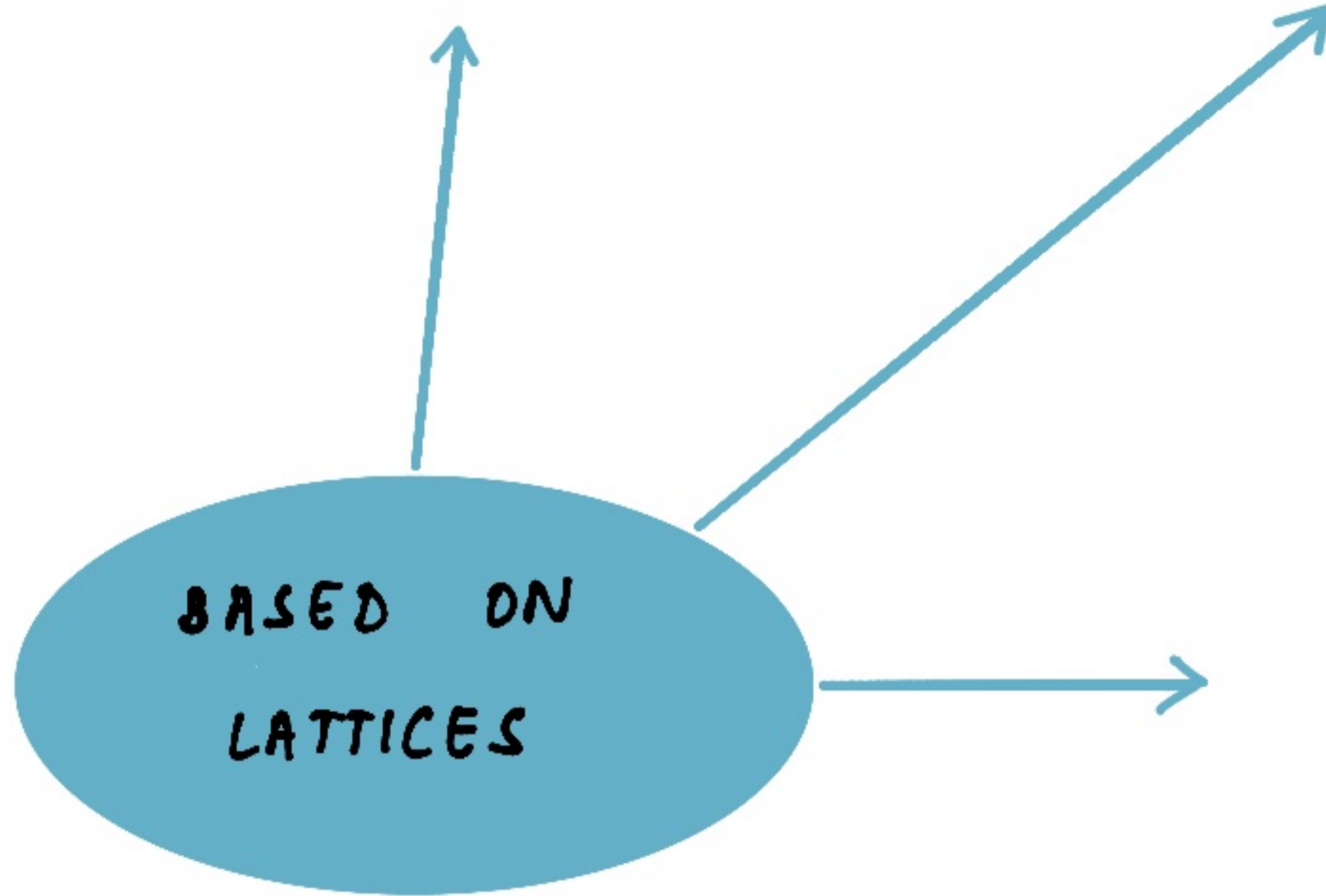
INTERESTINGLY



CRYSTALS-KYBER

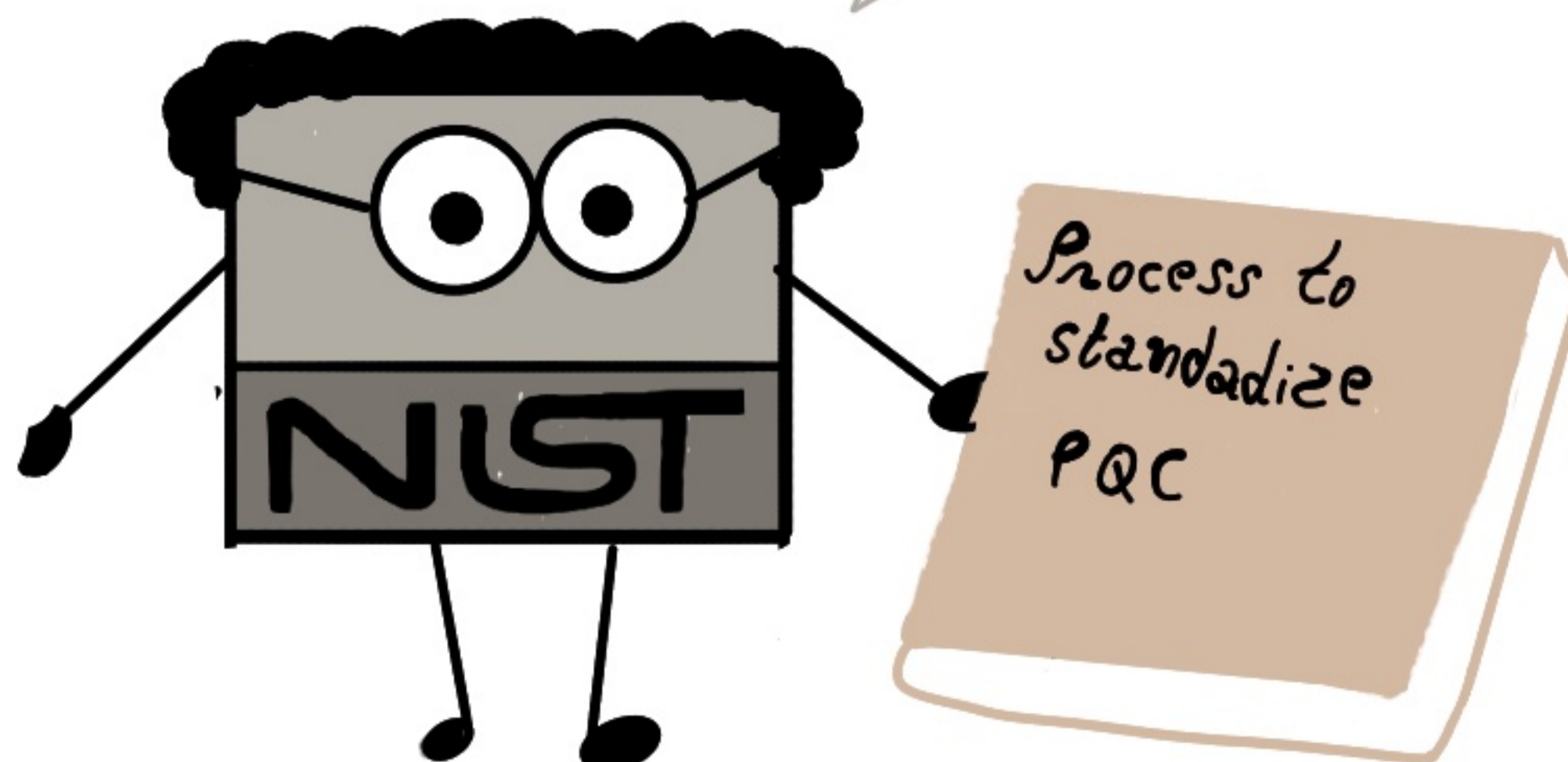


CRYSTALS-DILITHIUM



SPHINCS+

WE NEED ADDITIONAL DIGITAL SIGNATURES - BECAUSE 2 OF 3 ARE LATTICE BASED! ALSO SPHINCS+ DOESN'T PERFORM GREAT.



IN 2024



BIKE

HQC

HQC

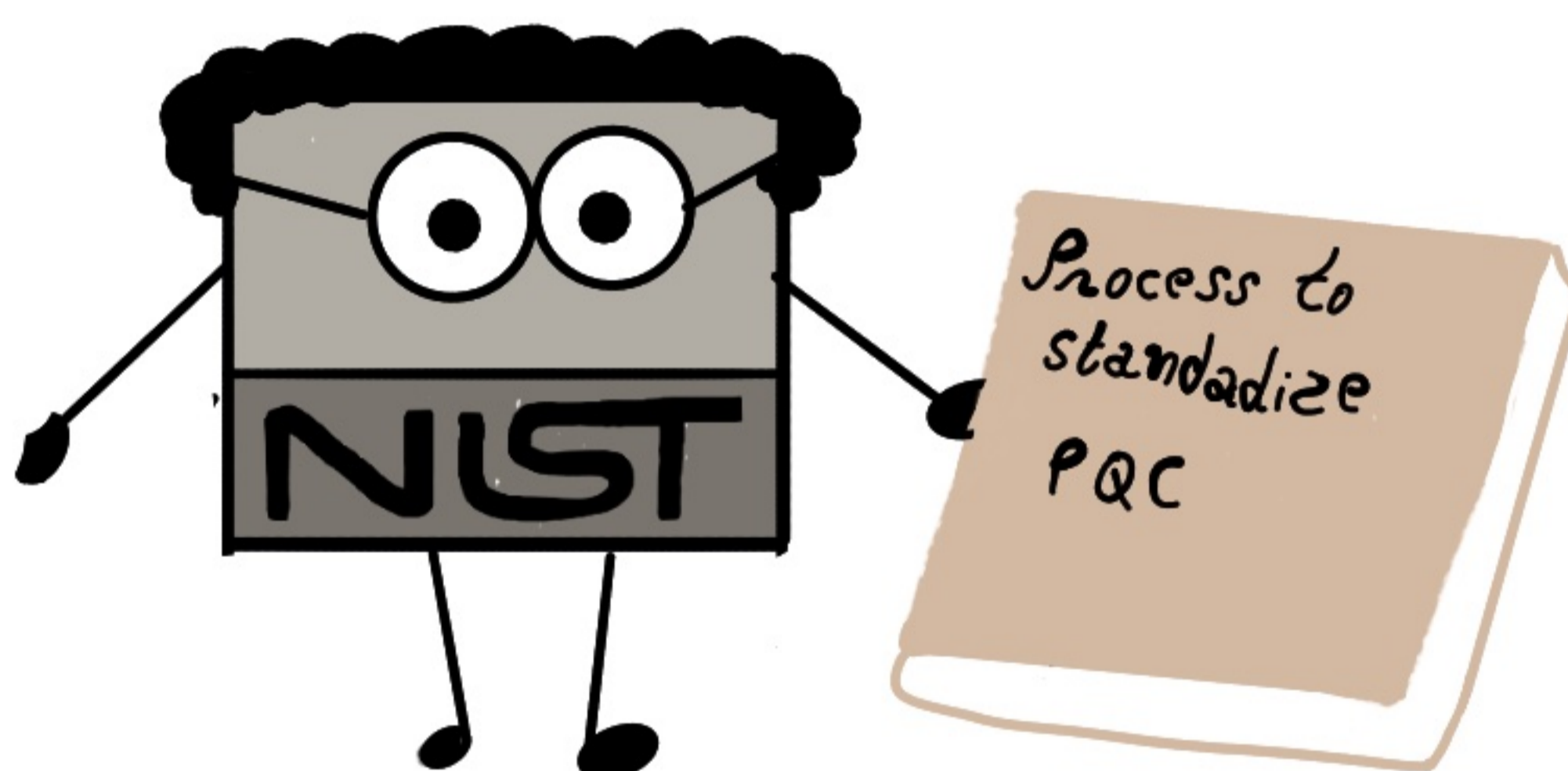
Classic McEliece

CLASSIC MCELIECE

BIT FLIPPING
KEY ENCAPSULATION

HAMMING
QUASI-CYCLE

NAMED AFTER
ROBERT J MCELIECE



WILL PRESENT
THEIR UPDATES
TO BE CONSIDERED

PART OF THE STANDARD FOR
KEY ENCAPSULATION MECHANISM
AT THE

5TH NIST PQC STANDARDISATION CONFERENCE

STANDARDS ORGANISATIONS

THESE ARE NAMES OF STANDARDS ORGANISATIONS THAT NIST ALSO WORKS WITH.

● ASC X9

● IEEE

● IETF

● ETSI

● PQCRYPTO

● SAFECRYPTO

● ISO/IEC JTC

THEY FREQUENTLY PUBLISH GUIDELINES AND PAPERS.



AS AN ASIDE, SOME COUNTRIES LIKE GERMANY, JAPAN, CHINA, RUSSIA, SOUTH KOREA ETC HAVE THEIR OWN STANDARDS.

QUANTUM-SAFE SCHEMES

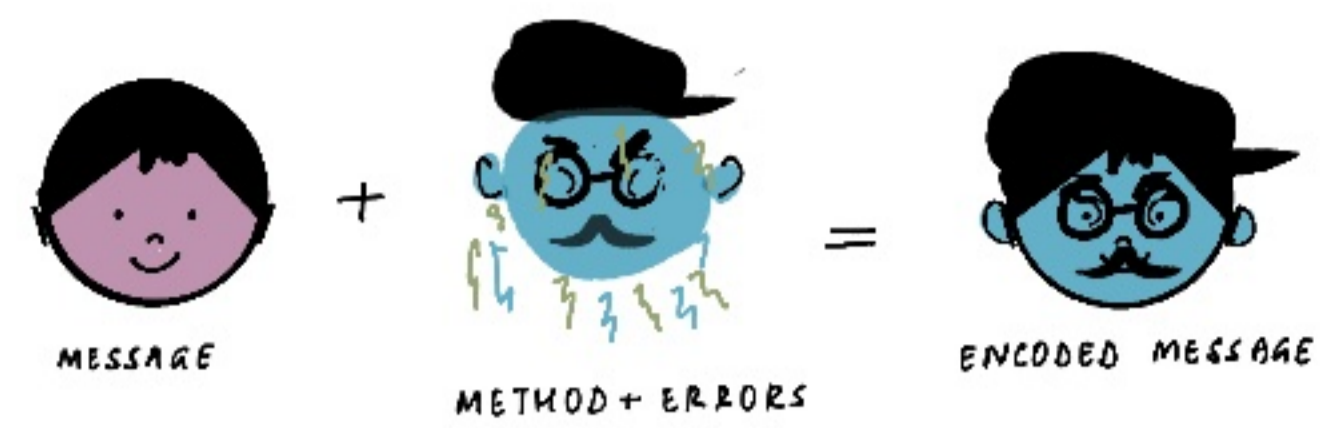
HASH BASED



SPHINCS +

USE HASH FUNCTIONS
FOR SIGNATURES

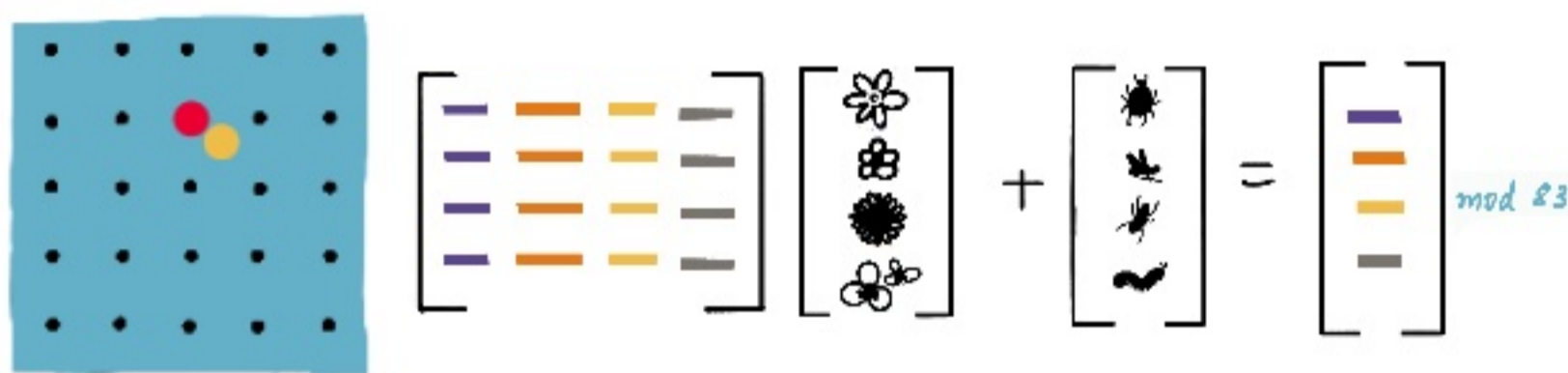
CODE BASED



MC ELIECE

USES ERROR CORRECTING
CODES FOR ENCRYPTION

LATTICE BASED



KYBER

USES LATTICE BASED
NP HARD PROBLEMS

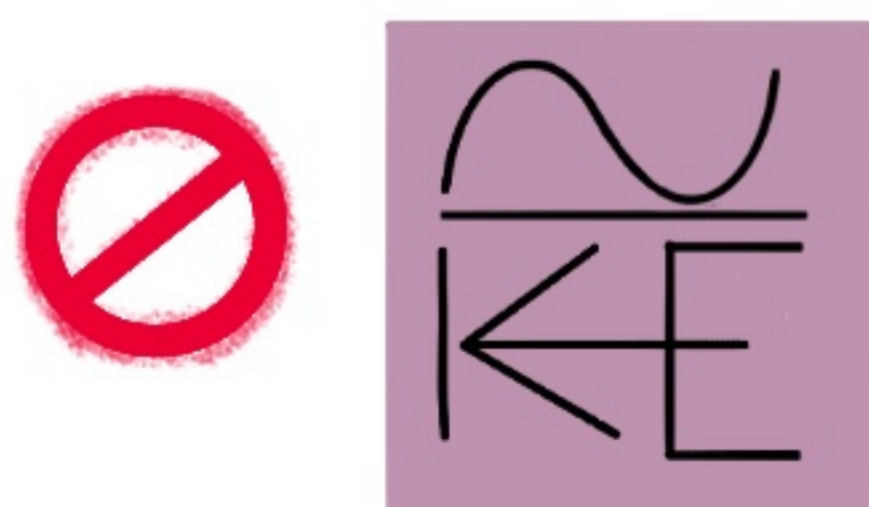
SYMMETRIC KEYS



AES

IDEAS THAT TRIED - AND FAILED - TO BE SECURE

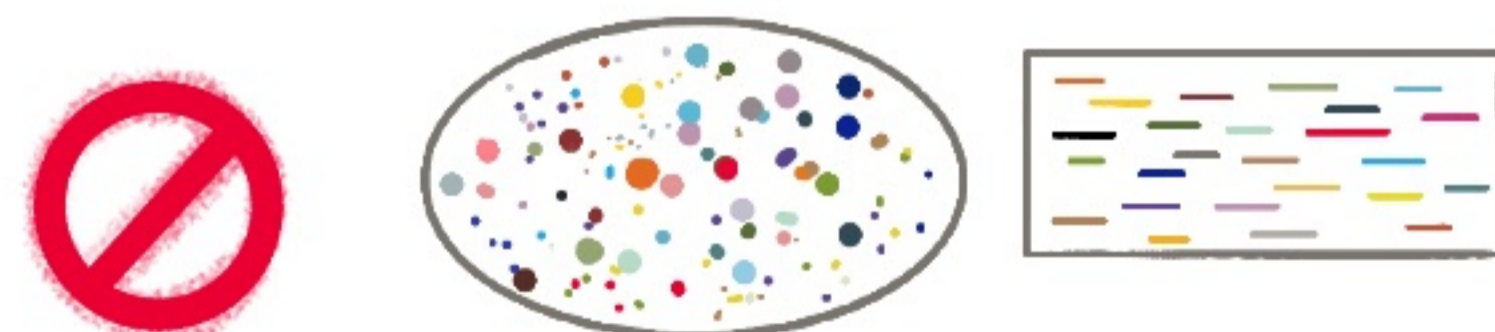
ISOGENY BASED



MATHEMATICAL FUNCTIONS
IN AN ELLIPTIC CURVE

SIKE

MULTIVARIATE



USES EQUATIONS WITH
MULTIPLE VARIABLES

RAINBOW

DEVELOPING AN INTUITION

IN THE NEXT FEW PAGES, WE WILL VISIT SOME OF THE CANDIDATE ALGORITHMS TO GAIN AN UNDERSTANDING OF THE PRINCIPLES/IDEAS BASED ON WHICH THEY WORK.

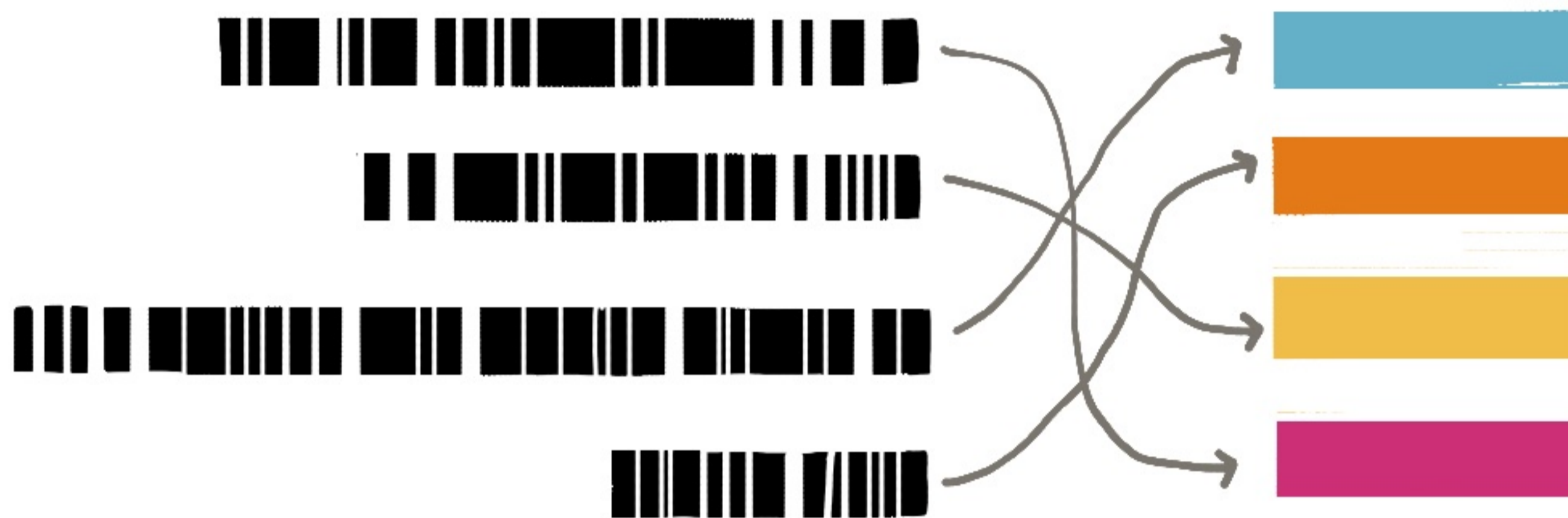
WHILE WE WILL NOT BE GOING INTO STEP-BY-STEP WALKTHROUGH OF THE ALGORITHMS, I HOPE VERY MUCH THAT THE READER WILL GET AN APPRECIATION FOR THE DESIGN OF THE ALGORITHM AND WHAT MAKES IT HARD.

HASH BASED CRYPTOGRAPHY

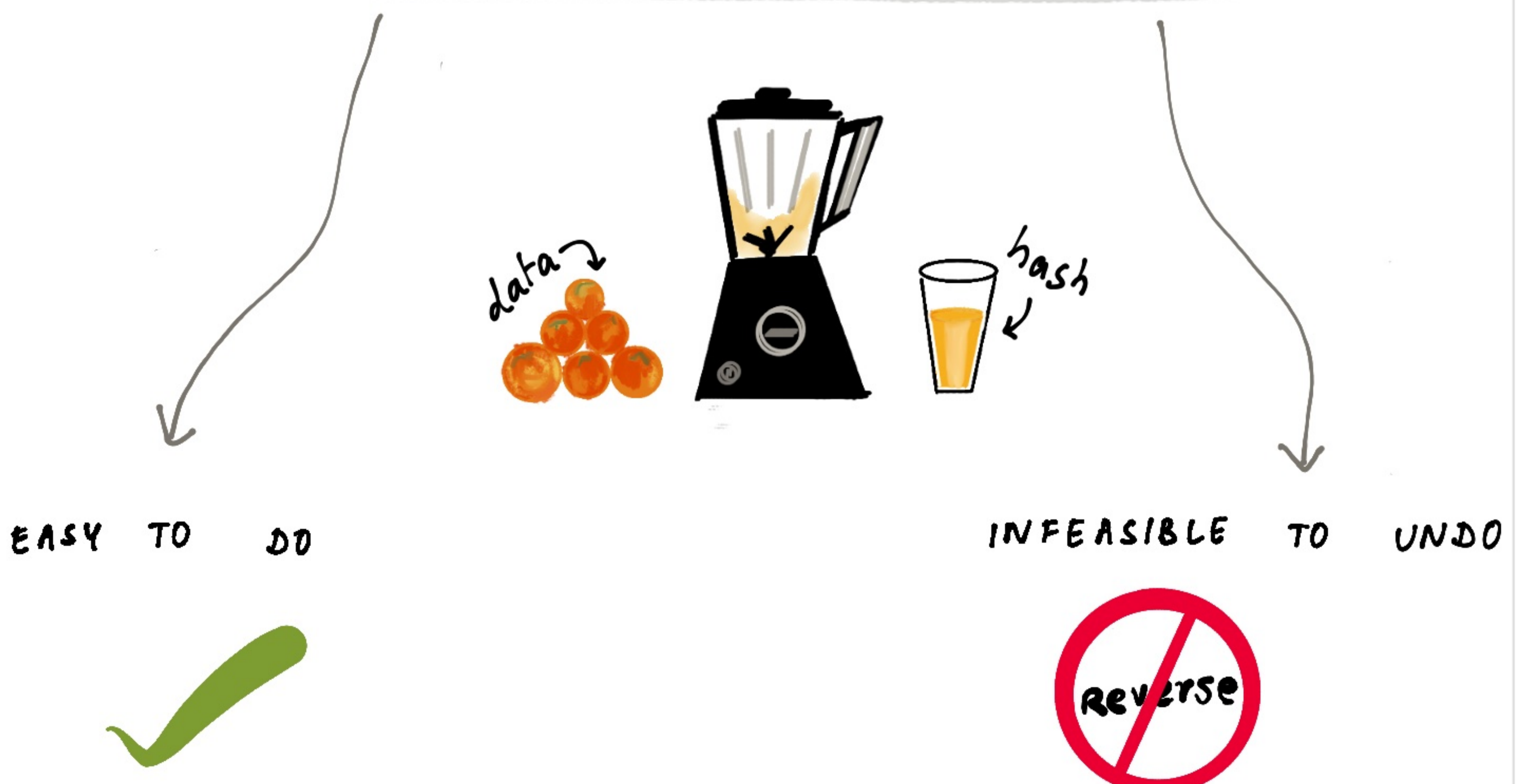
HASH BASED CRYPTOSYSTEMS

HASH BASED ENCRYPTION SCHEMES USE HASH FUNCTIONS AS THE BASIS FOR CREATING DIGITAL SIGNATURES

A HASH FUNCTION
MAPS DATA TO A FIXED LENGTH VALUE

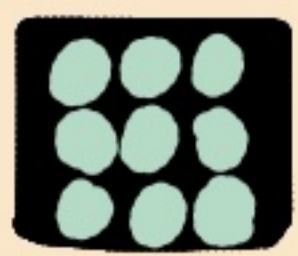


THE HASH IS A
ONE WAY MATHEMATICAL FUNCTION



HASH: SIMPLE EXAMPLE

A made-up hashing algorithm: (Data is 41)



square it

$$41 \times 41 = 1681$$



split numbers into
2 groups

$$16 \quad 81$$



Add the 2 groups

$$16 + 81 = 97$$

THIS IS EASY TO CALCULATE

HOWEVER, IT IS
NEAR IMPOSSIBLE
TO WORK OUT
WHICH EXACT NUMBERS
ADD UP TO 97

$$\text{Hash}(59) = 115$$

$$\text{Hash}(40) = 16$$

$$\text{Hash}(42) = 81$$

ANY OTHER NUMBER
PUT THROUGH THIS
ALGORITHM WILL
RESULT IN A COMPLETELY
DIFFERENT HASH

HASHES ARE USED
FOR VERIFYING INTEGRITY

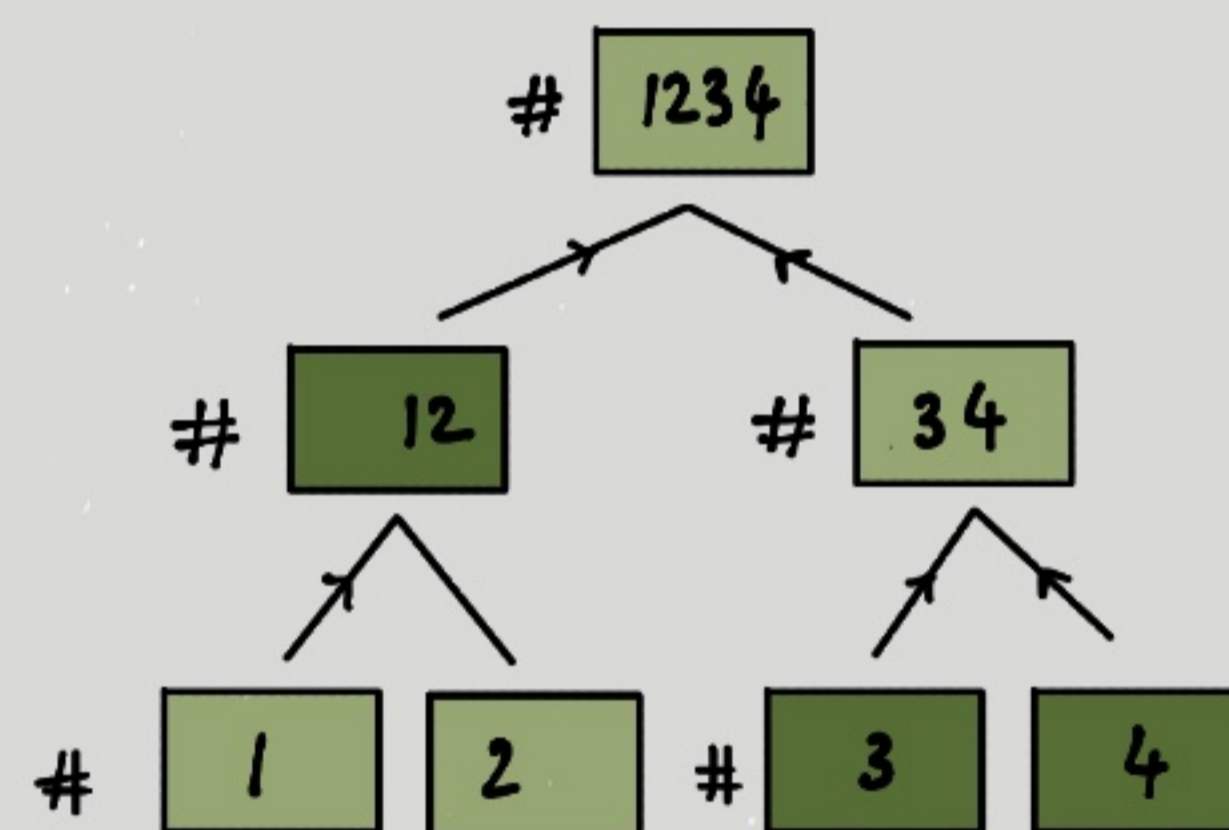
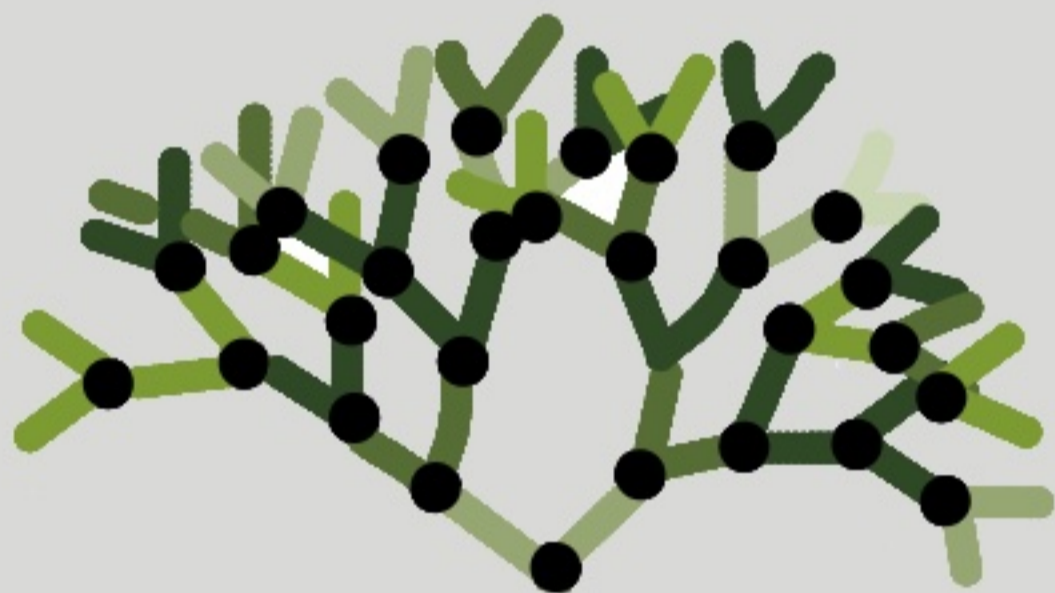
HASH: NIST CANDIDATE



SPHINCS+

SPHINCS+ IN THE NIST LIST IS A HASHING ALGORITHM THAT CENTRES AROUND MERKLE TREES.

A MERKLE TREE IS A TREE OF HASH VALUES.



A NODE IS THE HASH OF ITS CHILD NODES.

HASH BASED ALGORITHMS CAN BE USED IN DIGITAL SIGNATURES.

THE ABSENCE OF STRUCTURE/PATTERNS IN THE HASH MAKES IT HARD FOR QUANTUM COMPUTERS TO EXPLOIT THEM

BENEFITS

FAST VERIFICATION SPEEDS

WELL UNDERSTOOD BUILDING BLOCKS

CONSIDERATIONS

RELATIVELY LARGE SIGNATURE SIZES

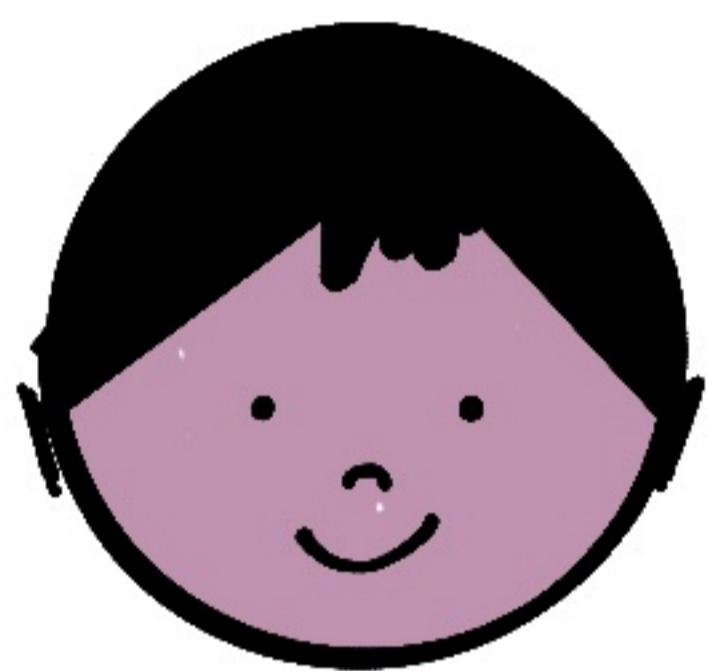
DILITHIUM & FALCON PERFORM BETTER

**CODE BASED
CRYPTOGRAPHY**

CODE-BASED CRYPTOSYSTEMS

CODE BASED ENCRYPTION SCHEMES ARE BASED ON THE DIFFICULTY OF DECODING ERROR CORRECTING CODES

AN ERROR CORRECTING CODE ENCODES MESSAGES. SO, EVEN WHEN THE BITS ARE FLIPPED, THEY CAN BE SPOTTED AND RECOVERED!



MESSAGE

+



METHOD + ERRORS

=



ENCODED MESSAGE

DECODING A CERTAIN TYPE OF CODE - A LINEAR CODE - IS AN NP COMPLETE PROBLEM.

On the Inherent intractability
of certain coding problems

by

E.R. Berlekamp
RJ McEliece
HCA Tilborg, van

Year - 1978

IMPLYING THAT
THERE IS NO
EFFICIENT
POLYNOMIAL-TIME
ALGORITHM TO
SOLVE IT

CODE: SIMPLE EXAMPLE

LET US TAKE A HIGHLY SIMPLIFIED EXAMPLE USING
HAMMING CODES TO SEE HOW ERROR CORRECTION WORKS



HAMMING (7,4)
CAN CORRECT A
SINGLE ERROR

Can you

Can you
hear me?

USES
PARITY BITS
TO BUILD IN
REDUNDANCY

oh Hello!



ODD PARITY IS ACHIEVED WHEN ADDING BITS

$$(1+0+1+\dots) \text{MOD } 2 = 1$$

EVEN PARITY IS ACHIEVED WHEN ADDING BITS

$$(1+0+1+\dots) \text{MOD } 2 = 0$$

CODE: SIMPLE EXAMPLE

- WE CONSIDER EVEN PARITY FOR THIS EXAMPLE

HAMMING (7,4) USES 4 DATA BITS AND 3 PARITY BITS

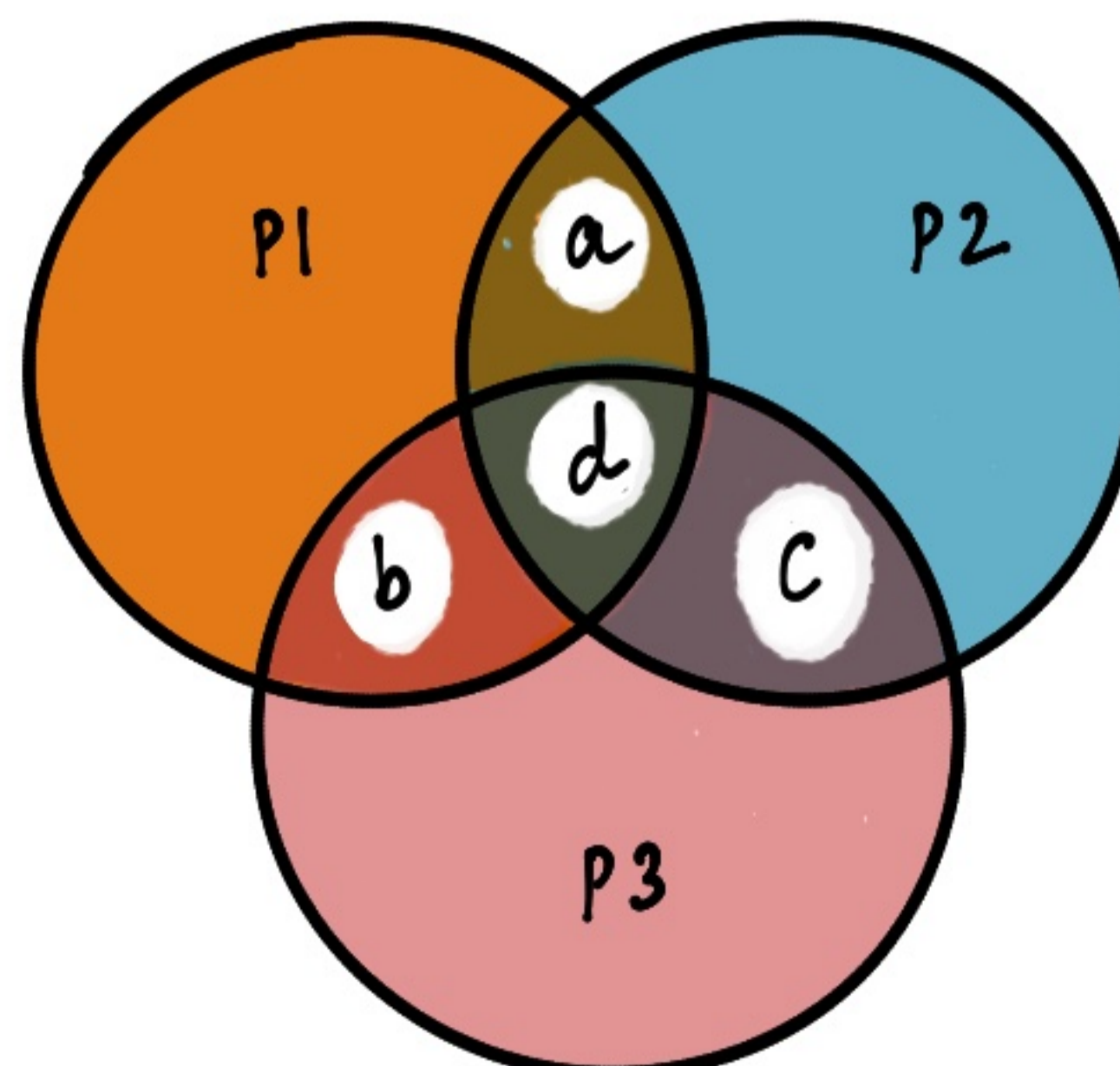


DATA BITS TO ENCODE IS 1011

PARITY BITS ARE PLACED IN POSITIONS 1, 2, 4, 8, ...

ENCODING →	P1	P2		P3			
POSITIONS →	1	2	3	4	5	6	7
POSITIONS (BINARY) →	001	010	011	100	101	110	111

DATA BITS ARE PLACED IN THE REST OF THE POSITIONS



CODE: SIMPLE EXAMPLE

FINDING VALUES
FOR THE PARITY BITS

		a		b	c	d
P1	P2	1	P3	0	1	1
1	2	3	4	5	6	7
001	010	011	100	101	110	111

P1 PARITY

$$P1 = a \oplus b \oplus d$$

INCLUDES POSITION NUMBERS WITH

1 AS THE LEAST SIGNIFICANT DIGIT

POSITIONS 1, 3, 5, 7 **P1 = 0**

P2 PARITY

$$P2 = a \oplus c \oplus d$$

INCLUDES POSITION NUMBERS WITH

1 AS THE 2ND LEAST SIGNIFICANT DIGIT

POSITIONS 2, 3, 6, 7 **P2 = 1**

P3 PARITY

$$P3 = b \oplus c \oplus d$$

INCLUDES POSITION NUMBERS WITH

1 AS THE 3RD LEAST SIGNIFICANT DIGIT

POSITIONS 4, 5, 6, 7 **P3 = 0**

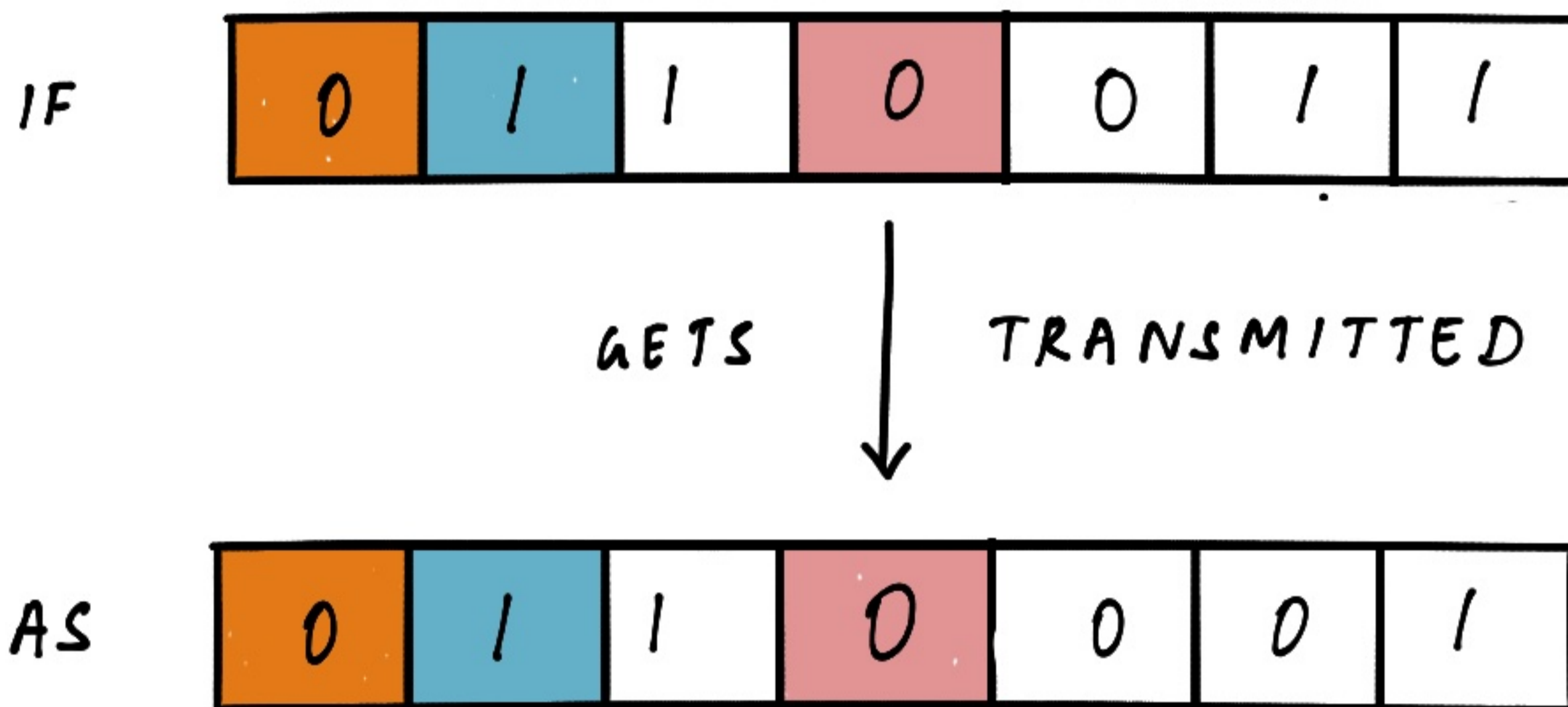
ENCODING IS :

0	1	1	0	0	1	1
---	---	---	---	---	---	---

CODE: SIMPLE EXAMPLE

ERROR CORRECTION

P1	P2	a	P3	b	c	d
1	2	3	4	5	6	7
001	010	011	100	101	110	111



P3 IS POSITIONS \rightarrow 4 5 6 7 \rightarrow 0001 \rightarrow PARITY = 1

P2 IS POSITIONS \rightarrow 2 3 6 7 \rightarrow 1101 \rightarrow PARITY = 1

P1 IS POSITIONS \rightarrow 1 3 5 7 \rightarrow 0101 \rightarrow PARITY = 0

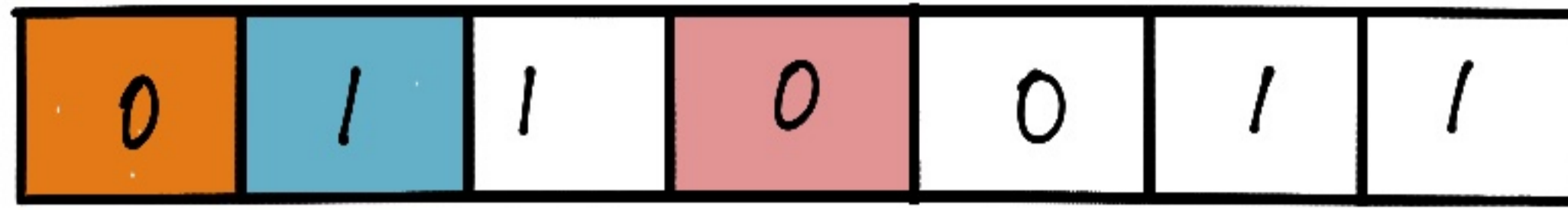
110 — IS POSITION 6 IN BINARY.

FLIP THE BIT IN POSITION 6 TO CORRECT ERROR

CODE : SIMPLE EXAMPLE

HAMMING CODES AND MATRICES

HAMMING CODES SUCH AS THIS



CAN ALSO BE GENERATED USING FORMULAE BASED ON MATRICES

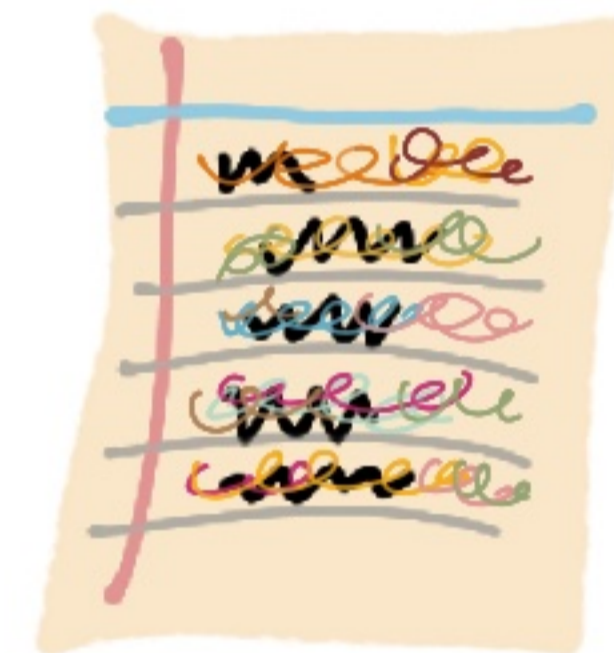
A MATRIX IS DATA ARRANGED IN ROWS AND COLUMNS

ENCODING A MESSAGE AS A MATRIX

$$\begin{matrix} \boxed{\text{DATA}} & \cdot & \boxed{\text{GENERATOR MATRIX}} & = & \boxed{\text{ENCODING}} \\ 1 \times 4 & & 4 \times 7 & & 1 \times 7 \end{matrix}$$



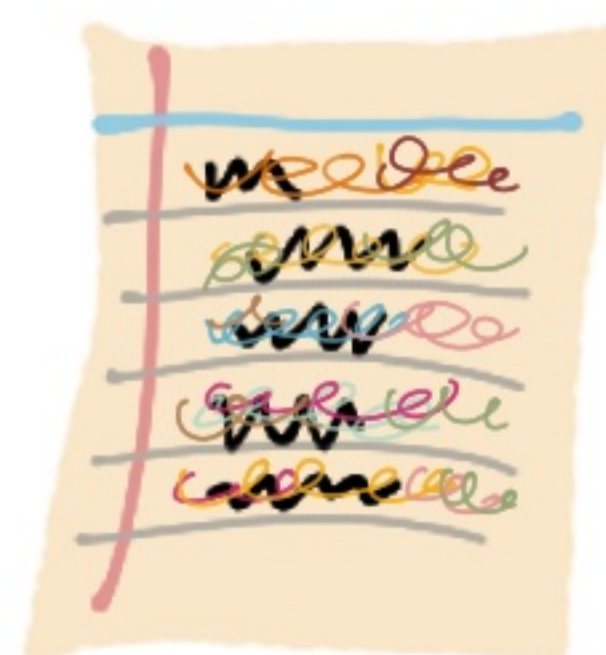
MATHEMATICALLY DERIVED



DECODING THE RECEIVED MESSAGE USING MATRICES

$$\begin{matrix} \boxed{\text{PARITY CHECK MATRIX}} & \cdot & \boxed{\text{ENCODING}} & = & \boxed{\text{POINTS TO THE}} \\ 3 \times 7 & & 7 \times 1 & & \text{ERROR POSITION} \\ & & & & 3 \times 1 \end{matrix}$$

MATHEMATICALLY DERIVED



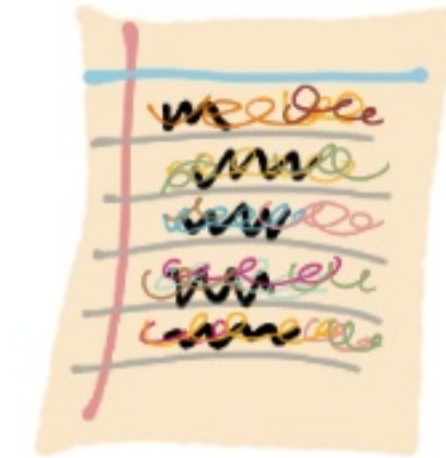
CODE : SIMPLE EXAMPLE

ENCODING A MESSAGE AS A MATRIX

$$\begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$



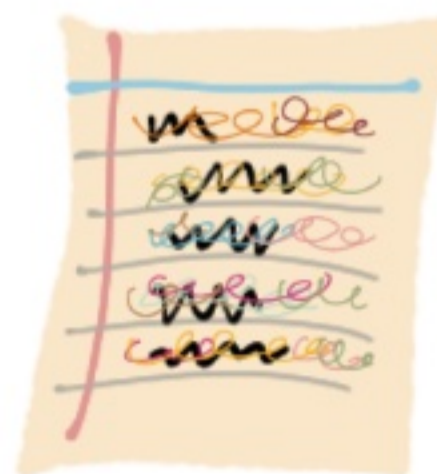
MATHEMATICALLY DERIVED



DECODING THE RECEIVED MESSAGE USING MATRICES

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

MATHEMATICALLY DERIVED



THIS POINTS TO A COLUMN NUMBER - i.e. THE POSITION OF THE BIT THAT NEEDS TO BE FLIPPED!

NOTICE THE ERROR?

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ INDICATES NO ERRORS

CODE: APPLICATION

My secret key is made of 3 matrices $[P]$ $[G]$ and $[S]$

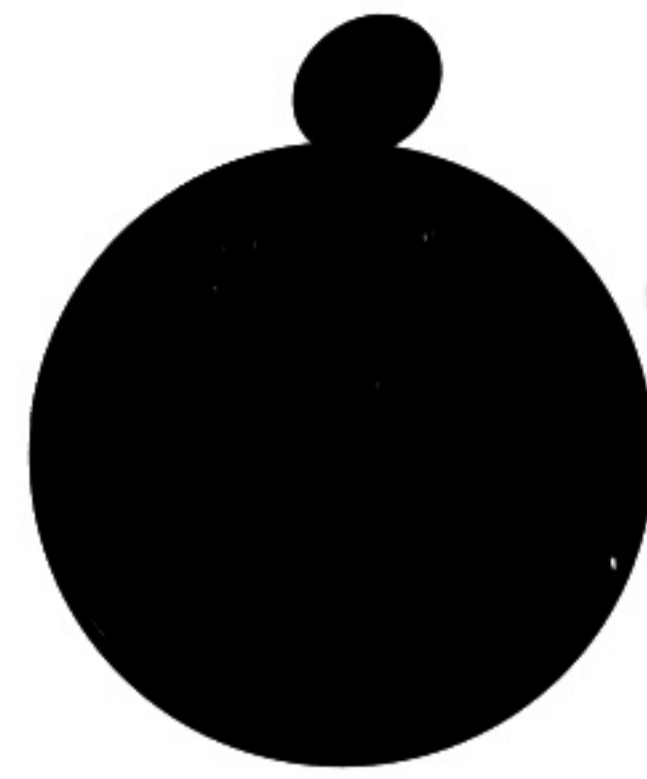
My public key is the product of $[P][G][S]$

$$[P][G][S] = \hat{G}$$

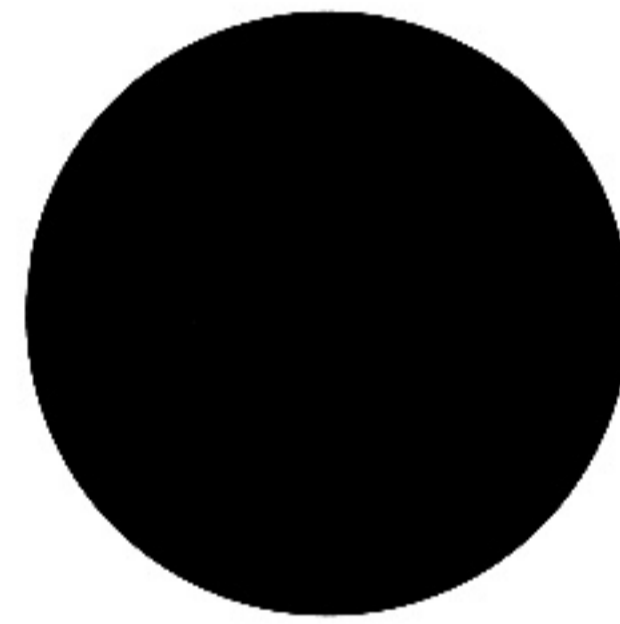
To encrypt message m send $\hat{G} \cdot m + \text{'errors'}$

I will struggle to invert the matrices or decode the message without $[P][G][S]$

Used Goppa codes - not Hamming codes



ALICE



SENDER: BOB



EAVESDROPPER: EVE

THIS ROUGHLY IS THE BASIS FOR THE MCELIECE CRYPTOSYSTEM
MCELIECE IS IN ROUND 4 FOR CONSIDERATION WITH NIST.

MKPC : NIST CANDIDATE



CLASSIC MCELIECE IS NOT YET IN THE STANDARDS LIST - BUT IS BEING CONSIDERED AS A KEY ENCAPSULATION MECHANISM.

BENEFITS

UNBROKEN FOR 40 YEARS

FAST ENCRYPTION/DECRYPTION

CONSIDERATIONS

LARGE KEY SIZES

NOT SUITABLE IN CASE OF LIMITED BANDWIDTHS

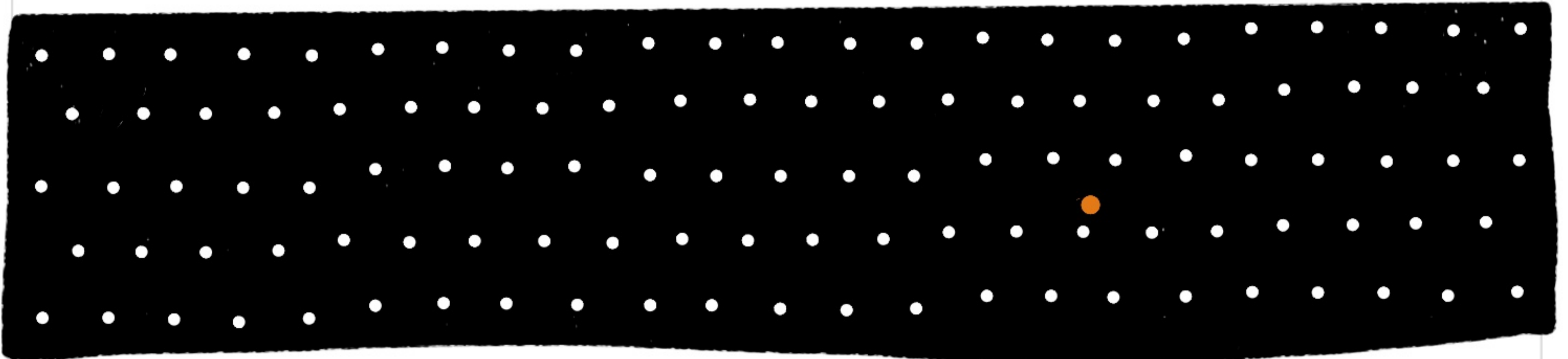
LATTICE BASED
CRYPTOGRAPHY

LATTICE BASED SCHEMES

LATTICE BASED CRYPTOSYSTEMS USE WELL STUDIED NP-HARD LATTICE PROBLEMS SUCH AS...

... CLOSEST VECTOR PROBLEM

IN AN INFINITE GRID OF DOTS IN HUNDREDS OF DIMENSIONS PICK A POINT IN SPACE AND FIND THE NEAREST DOT.



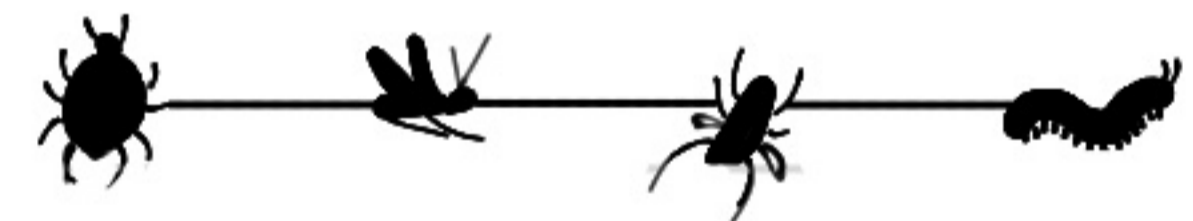
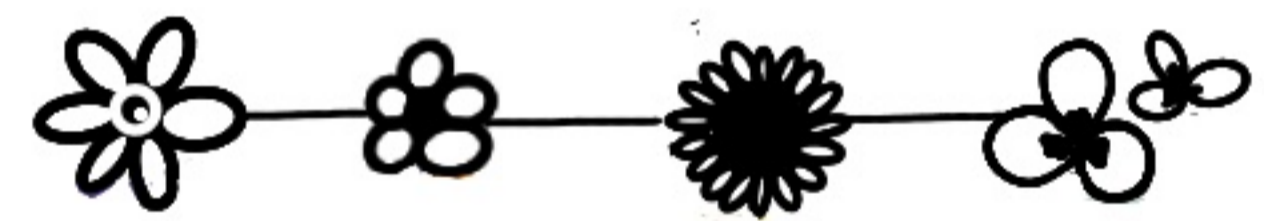
... LEARNING WITH ERRORS

TAKE A SYSTEM OF EQUATIONS

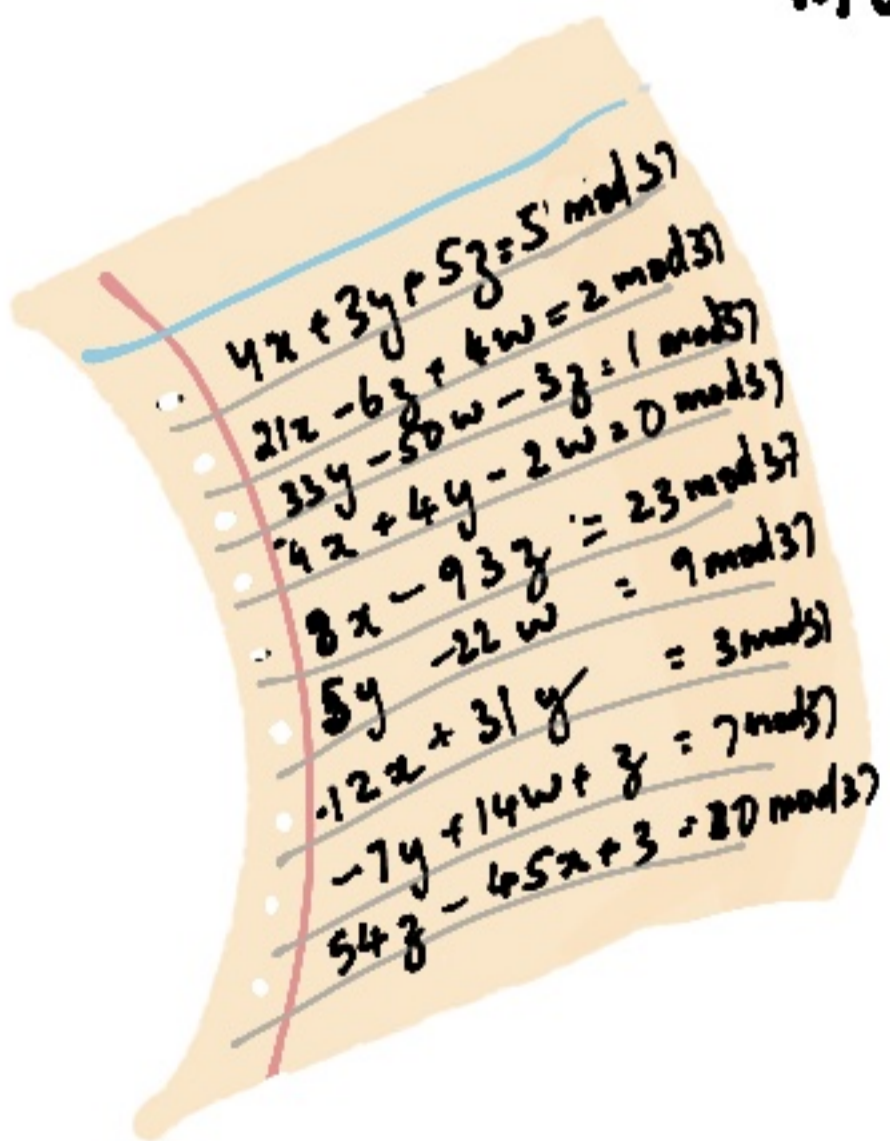
CONSTRUCTED USING SOME SECRET KEYS








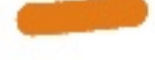




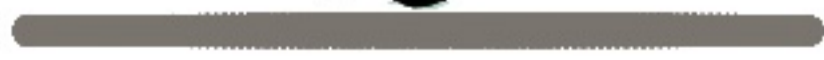



WITH SECRETLY ADDED ERRORS

MOD A PRIME NUMBER



MOD 83

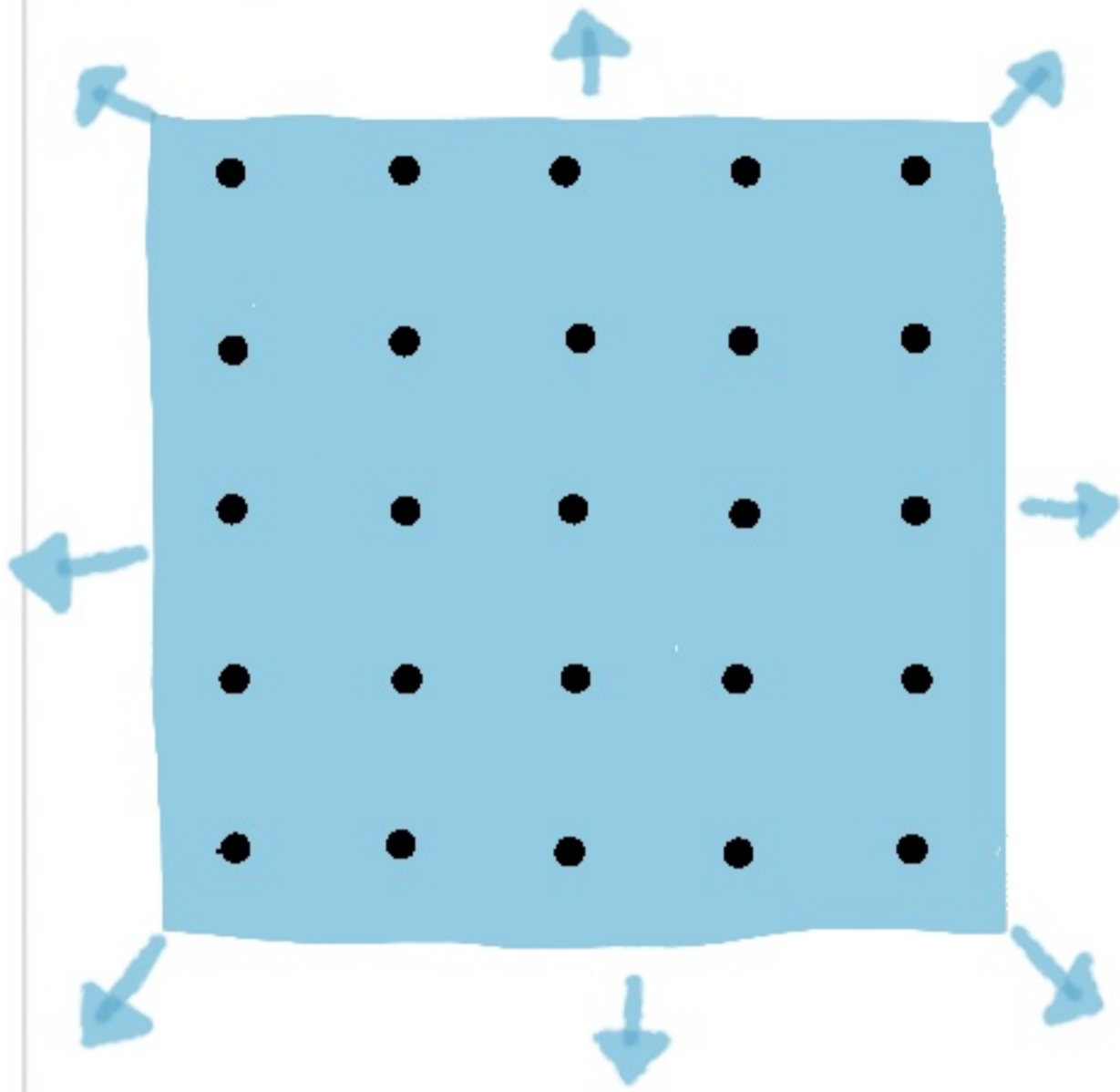


		+		=		mod 83
		+		=		mod 83
		+		=		mod 83
		+		=		mod 83

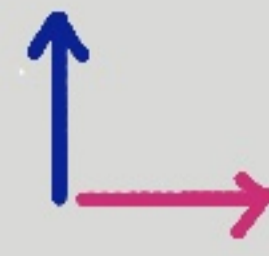
TURNS OUT THEY ARE BOTH SOME VERSION OF EACH OTHER AND ARE HARD FOR QUANTUM COMPUTERS TO SOLVE.

CLOSEST VECTOR PROBLEM

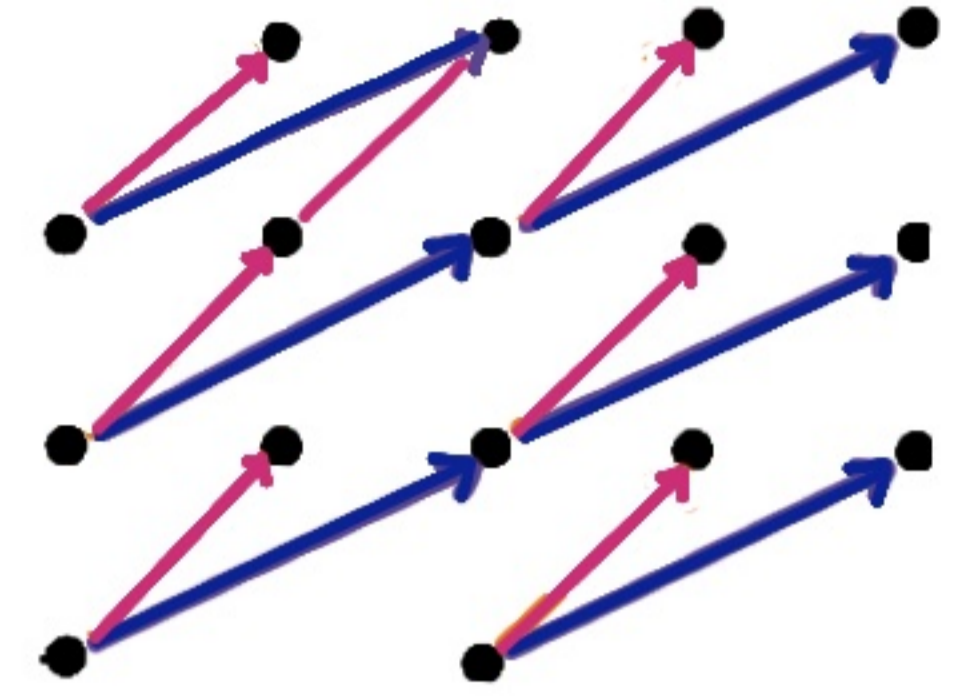
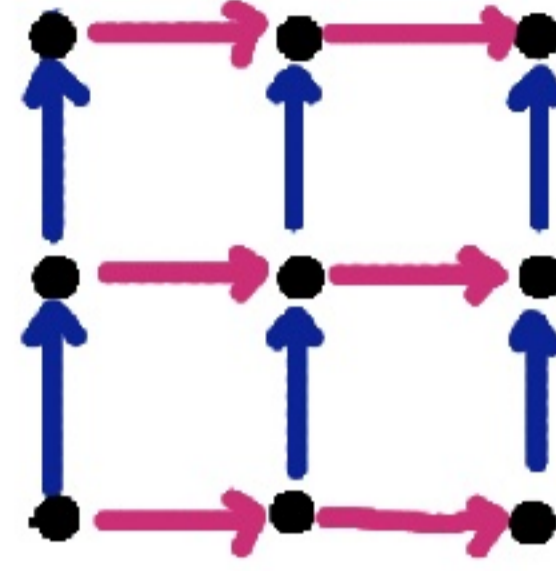
LATTICE :
A GRID OF 'DOTS'



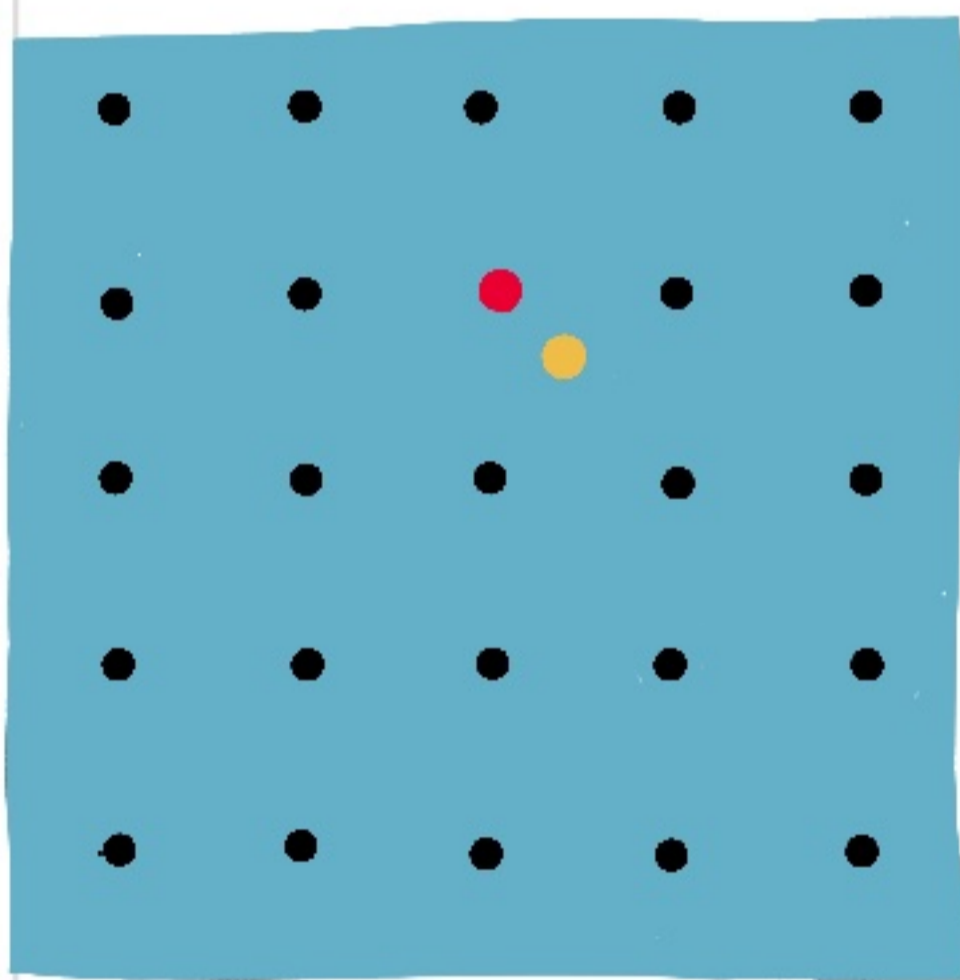
COMPOSED
LIKE THIS



OR
LIKE THIS



PICK 2 POINTS




THE YELLOW DOT ●

A POINT SLIGHTLY
OFF THE LATTICE

THE RED DOT ●

POINT ON THE
LATTICE CLOSEST TO
THE YELLOW DOT ●

MATHEMATICALLY, GIVEN THE DEFINITION OF A LATTICE 
(ONLY IMAGINE THE LATTICE IN HUNDREDS OF DIMENSIONS)
AND THE YELLOW DOT ● IT IS HARD TO FIND THE RED DOT ●

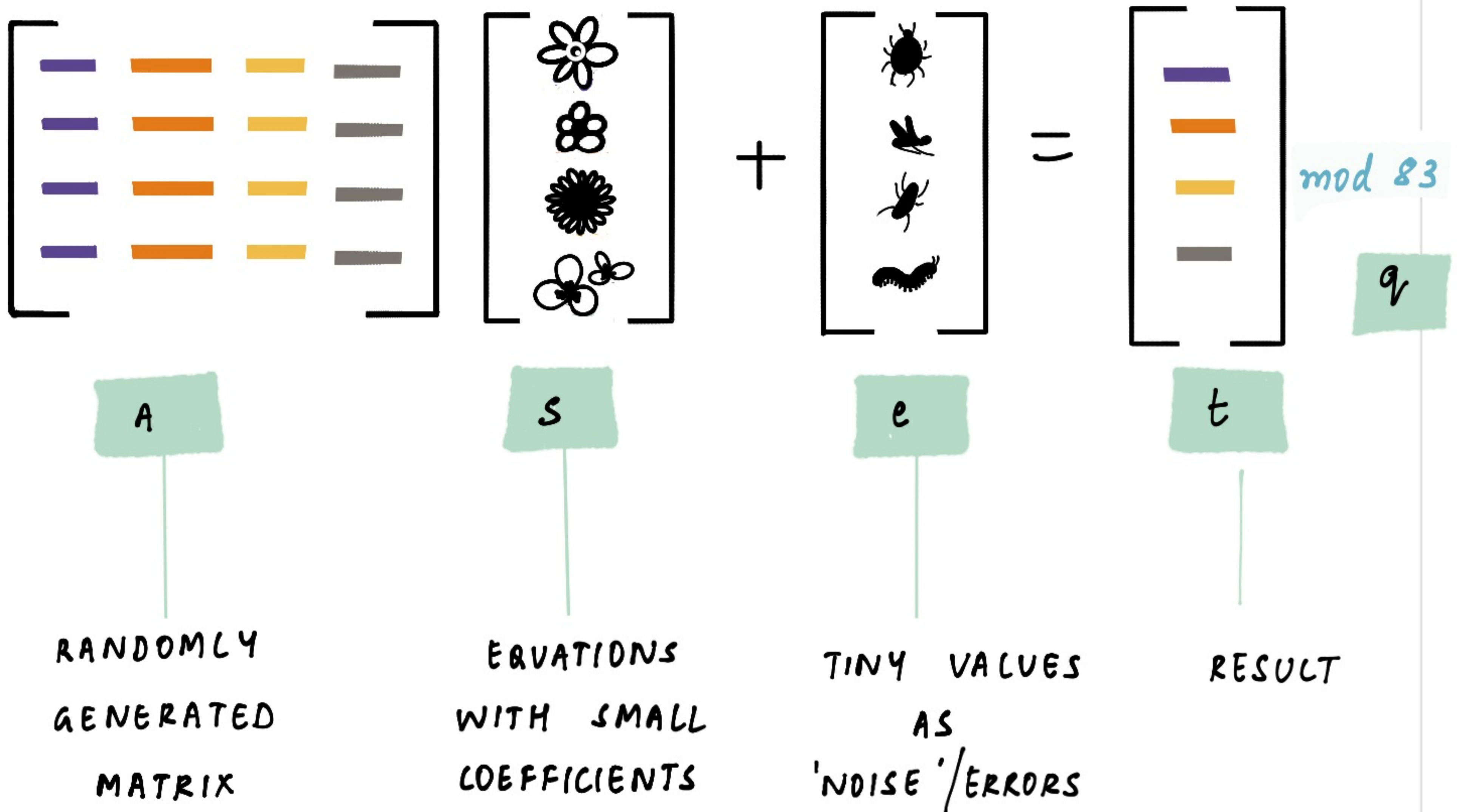
PUBLIC KEY



SECRET KEY ●

MODULE LEARNING WITH ERRORS

THE SYSTEM OF EQUATIONS WE SAW EARLIER CAN BE COMPACTLY ARRANGED INTO MATRIX FORMAT AS BELOW



$$A \cdot s + e = t$$

PUBLIC INFORMATION

A, t, q

PRIVATE INFORMATION

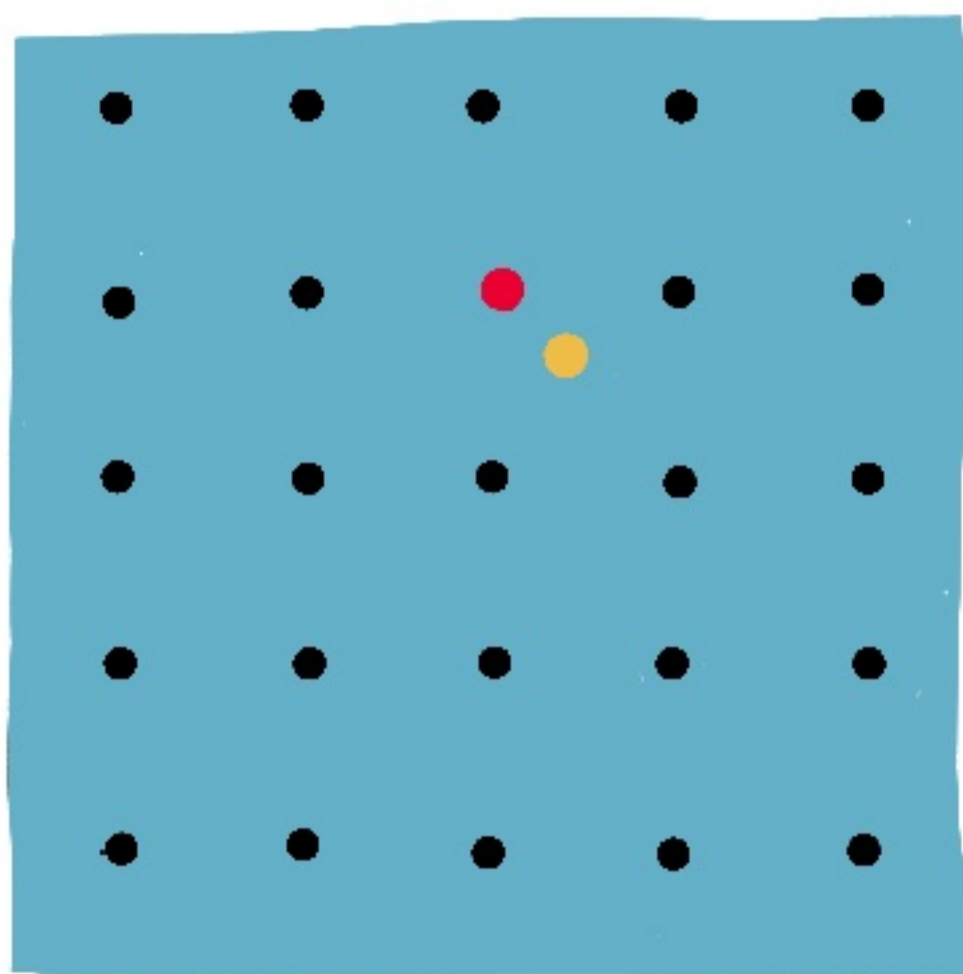
s, e

RETRIEVING THE SECRET **S** IS INCREDIBLY HARD IN THE PRESENCE OF **e** AND WHEN DIMENSIONS OF **A** ARE LARGE

THE NAME MODULE LEARNING WITH ERRORS IS THE NAME CHOSEN BY MATHEMATICIANS FOR THIS PROBLEM. SIGH

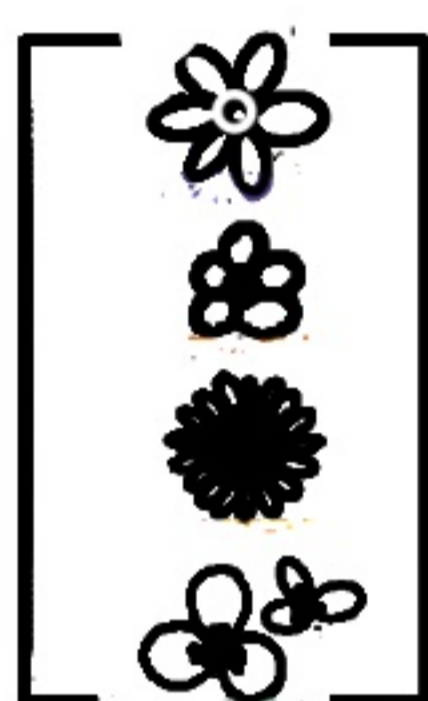
JOINING THE DOTS

WHAT IS THE CONNECTION BETWEEN THE CLOSEST VECTOR PROBLEM AND LEARNING WITH ERRORS?



A

DESCRIBES THE LATTICE



S

THE LOCATION OF THE SECRET POINT

● RED DOT



mod 83

t

IS THE PUBLIC POINT

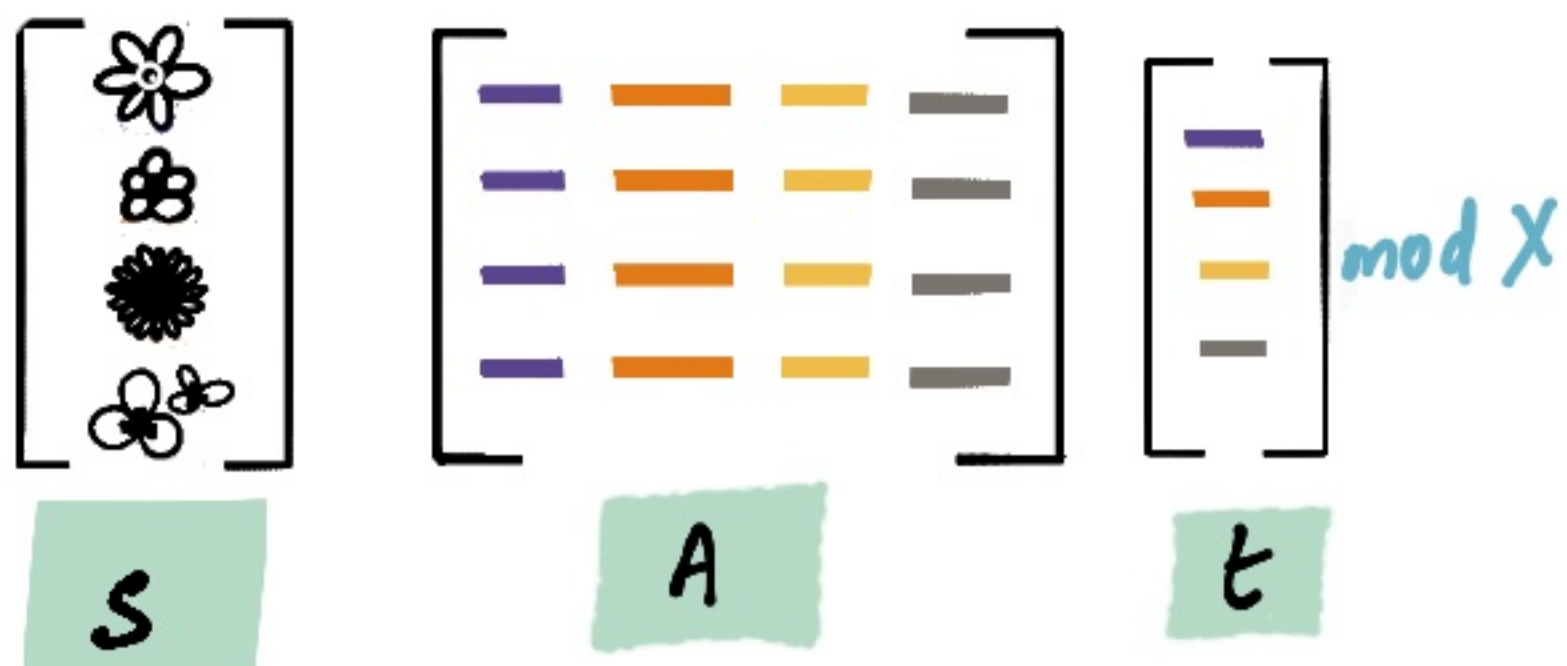
● YELLOW DOT

ARE ALL EQUATIONS IN POLYNOMIAL FORM

e.g. $5x^6 - 11x^5 - x^4 - 26x^3 + x + 5 = -5$

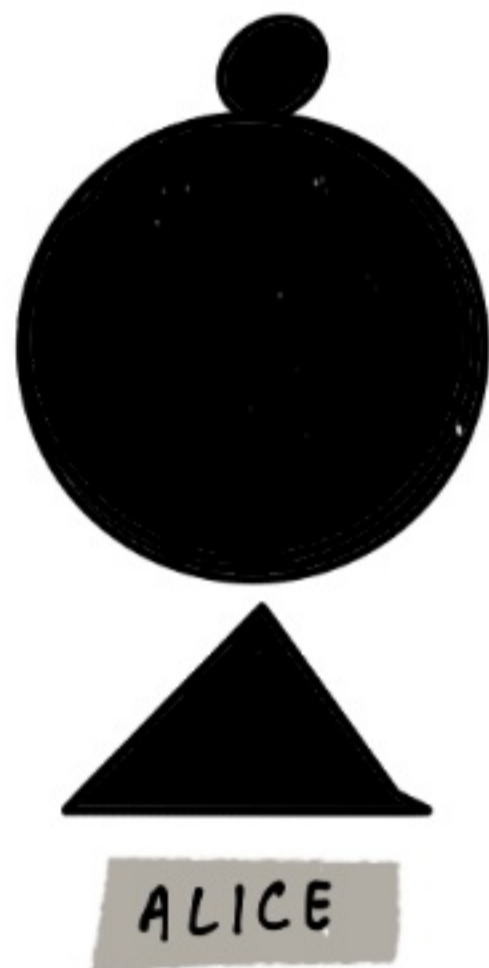
ARRANGED IN A MATRIX

LATTICE: SIMPLE EXAMPLE

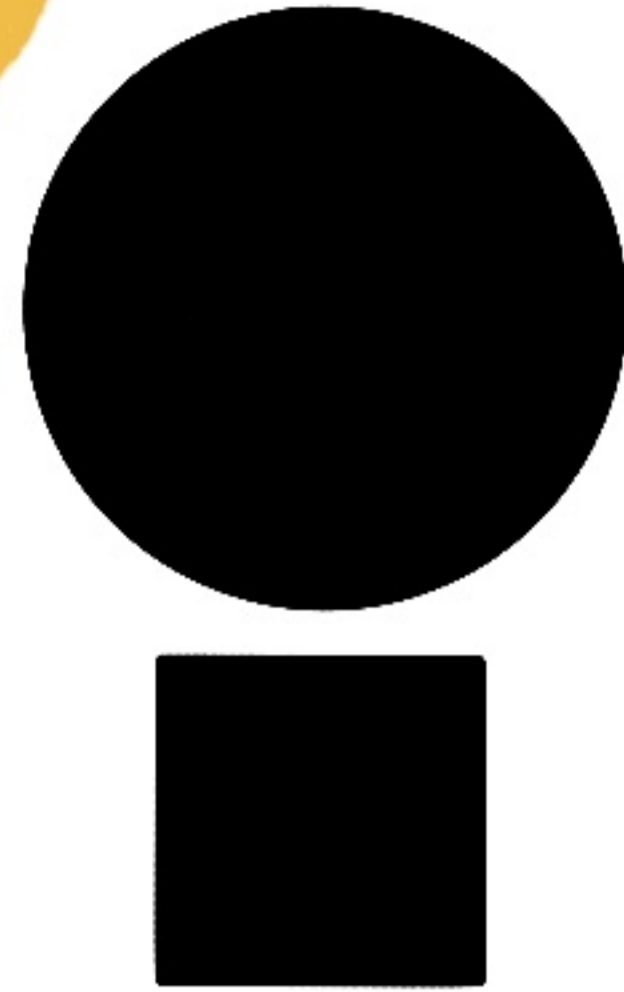


BOB HAS A MESSAGE M

10 11



ALICE HAS PUBLIC KEYS
A E



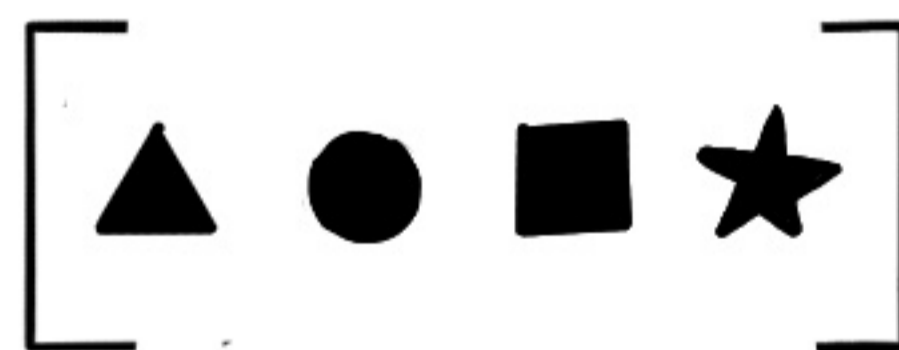
SENDER: BOB

TO ENCRYPT BOB MUST

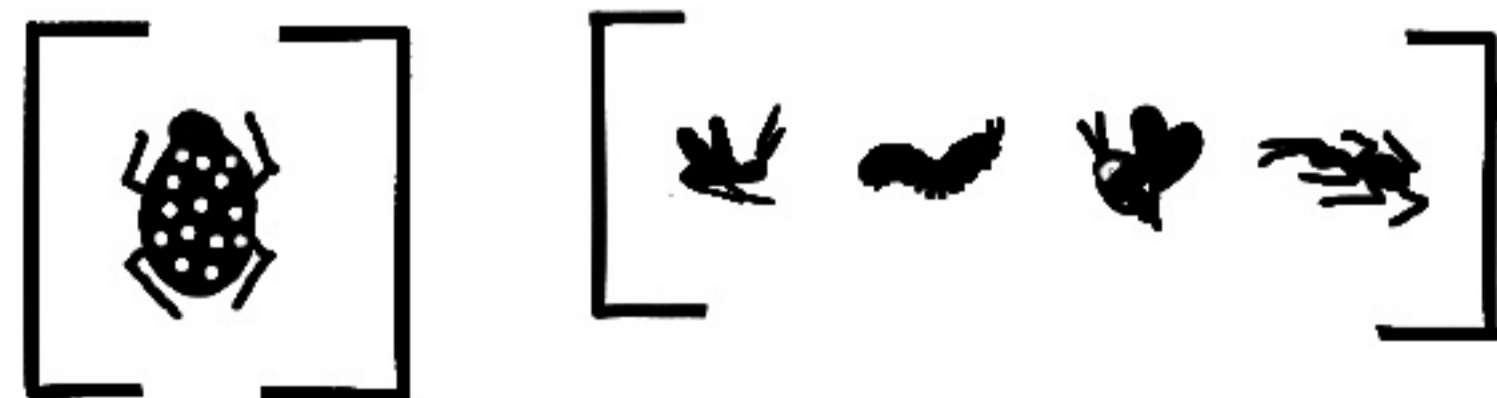
● CONVERT HIS BINARY MESSAGE TO POLYNOMIALS

DECIMAL	BINARY	POLYNOMIAL
1	001	$0 + 0 + 1 \rightarrow 1$
2	010	$0 + x + 0 \rightarrow x$
3	011	$0 + x + 1 \rightarrow x + 1$
4	100	$x^2 + 0 + 0 \rightarrow x^2$
5	101	$x^2 + 0 + 1 \rightarrow x^2 + 1$
7	111	$x^2 + x + 1$
10	1010	$x^3 + 0 + x + 1$

● CHOOSE A RANDOM MATRIX γ



● CHOOSE TWO OTHER SMALL 'ERRORS'



LATTICE: SIMPLE EXAMPLE

ENCRYPTION

BOB SENDS V AND U

$$V = \left[\begin{array}{c} \triangle \\ \bullet \\ \blacksquare \\ \star \end{array} \right] \left[\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right] + \left[\begin{array}{c} \text{bug} \end{array} \right] + 1011$$

γ ——— t ——— e_1 ——— m

$$U = \left[\begin{array}{c} \triangle \\ \bullet \\ \blacksquare \\ \star \end{array} \right] \left[\begin{array}{cccc} \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \end{array} \right] + \left[\begin{array}{cccc} \text{bird} & \text{bird} & \text{bird} & \text{bird} \end{array} \right]$$

γ ——— A ——— e_2

DECRYPTION

ALICE FINDS MESSAGE $M = V - US$

$$M = \gamma t + e_1 + m - (\gamma A + e_2) S$$

$$= \gamma (AS + e) + e_1 + m - \gamma AS - e_2 S$$

$$= \cancel{\gamma AS} + \cancel{\gamma e} + e_1 + m - \cancel{\gamma AS} - \cancel{e_2 S}$$

VERY SMALL TERMS WITH e , e_1 , e_2 ARE IGNORED - LEAVING M

LATTICE: NIST CANDIDATE



CRYSTALS - KYBER

THE ILLUSTRATION IN THE PREVIOUS PAGE IS A SIMPLIFIED VERSION OF THE KYBER CRYPTOSYSTEM.

BENEFITS

BASED ON STRUCTURED LATTICES
- KNOWN HARD PROBLEM

GOOD PERFORMANCE & SECURITY

CONSIDERATIONS

AS WITH ANY SCHEME,
IT REQUIRES CAREFUL
IMPLEMENTATION

LONG TERM SECURITY
IMPLICATIONS UNCLEAR

**A PRC SCHEME THAT
HAS BEEN BROKEN**

THE BROKEN SCHEMES

SIKE - AN ISOGENY BASED SCHEME

SIKE IS ROUGHLY SIMILAR TO THE DIFFIE-HELLMAN KEY EXCHANGE DISCUSSED IN THE ILLUSTRATED GUIDE 'HOW TO TELL SECRETS' PUBLISHED BY THOUGHTWORKS.

SIKE WAS BROKEN IN 2022 AUGUST BY WOUTER CASTRYCK AND THOMAS DECRU OF BELGIUM IN UNDER AN HOUR WITH A SINGLE CORE. SIKE MADE IT TO ROUND 4.

RAINBOW - A MULTIVARIATE CRYPTOSYSTEM

RAINBOW IS A MULTIVARIATE CRYPTOSYSTEM. WHICH MEANS THE SECURITY LIES IN THE DIFFICULTY OF SOLVING A LARGE SYSTEM OF EQUATIONS WITH MANY VARIABLES.

RAINBOW WAS BROKEN IN 2022 USING A STANDARD LAPTOP OVER 53 HOURS BY WARD BEULLENS OF IBM, SWITZERLAND. RAINBOW MADE IT TO ROUND 3.

LET'S TAKE A QUICK LOOK AT RAINBOW'S APPROACH TO DESIGNING A QUANTUM-SAFE ALGORITHM - ALBEIT BROKEN.

MULTIVARIATE CRYPTOSYSTEMS

MULTIVARIATE PUBLIC KEY CRYPTOSYSTEMS ARE BASED ON THE HARDNESS OF SOLVING EQUATIONS WITH MULTIPLE VARIABLES

MQ challenge :
Hardness Evaluation
of Solving
Multivariate Quadratic Problems

T Yasuda, X Dahan, Y Huang
T Takagi, K Sakurai

eprint.iacr.org/2015/275.pdf

FOR EXAMPLE :

$$2x + 3 = 5$$

$$2x + 3xy + 5x^2 = 4 \pmod{3}$$

$$x^2 - 7xz + 2y^2 = 1 \pmod{3}$$

find
 x

find
 y

find
 z

SOLVING MEANS TO FIND THE VALUES OF THE VARIABLES x, y, z THAT WORKS FOR EACH EQUATION.

THE PROBLEM BECOMES HARDER WHEN THE NUMBER OF EQUATIONS AND VARIABLES DON'T MATCH EXACTLY.

MKPC

NOW, INSTEAD OF



3 VARIABLES



IMAGINE :



150 VARIABLES



2 EQUATIONS



50 EQUATIONS

IT TURNS OUT THAT **THIS** MANY VARIABLES AND POLYNOMIALS (EQUATIONS) **COULD** CREATE A PROBLEM TOO HARD — EVEN FOR A QUANTUM COMPUTER!

THE RESULT IS MULTIVARIATE PUBLIC KEY CRYPTOSYSTEM — **MKPC**

THIS SET OF EQUATIONS SERVES AS THE PUBLIC KEY

THE PRIVATE KEY IS A SUBSET OF THE VARIABLES
ODDLY NAMED OIL VARIABLES AND VINEGAR VARIABLES.

MKPC : SIMPLE EXAMPLE

TAKE A SINGLE EQUATION WITH VARIABLES a, b, c, d

$$a^2 + 3ab + 3ac + 2ad + b^2 + 6bc + 4bd = 25 \pmod{3}$$

POLYNOMIALS CAN BE EXPRESSED AS THE PRODUCT OF MATRICES. SO THE ABOVE BECOMES :

$$\begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 2 \\ 2 & 3 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

The trapdoor!

$$\begin{bmatrix} a+b+2c+d & 2a+b+3c+2d & a+3b & a+2b \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

NOTICE THAT BY THE DELIBERATE PLACEMENT OF THE ZERDES THE VARIABLES c, d DO NOT MULTIPLY WITH EACH OTHER IN THE ORIGINAL EQUATION.

IF YOU KNOW $a=1$ AND $b=1$ THEN THE EXPRESSION IS LINEAR - AND A BIT EASIER TO SOLVE THE EQUATION

$$1 + 3 + 3c + 2d + 1 + 6c + 4d$$

MKPC : SIMPLE EXAMPLE

$$a^2 + 3ab + 3ac + 2ad + b^2 + 6bc + 4bd = 25 \pmod{3}$$

$$\begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 2 \\ 2 & 3 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

The trapdoor!

a b

→ VINEGAR VARIABLES

- FORMS THE SECRET

c d

→ OIL VARIABLES



CHOOSE VARIABLES

100 VINEGAR & 50 OIL



CREATE 50 EQUATIONS

EACH WITH THE TRAPDOOR

DECRYPTION WILL INVOLVE
SOLVING 50 EQUATIONS
AND
EXACTLY 50 UNKNOWNNS!

THIS IS NOW BROKEN AND IT REMAINS TO BE SEEN
IF NEWER SCHEMES WILL APPEAR BASED ON THIS METHOD.

**MOVING TO
PRC**

FIRST STEPS

GATHER AN
EXPERT TRANSITION
TEAM



CONDUCT A
RISK ASSESSMENT
OF DATA & SYSTEMS



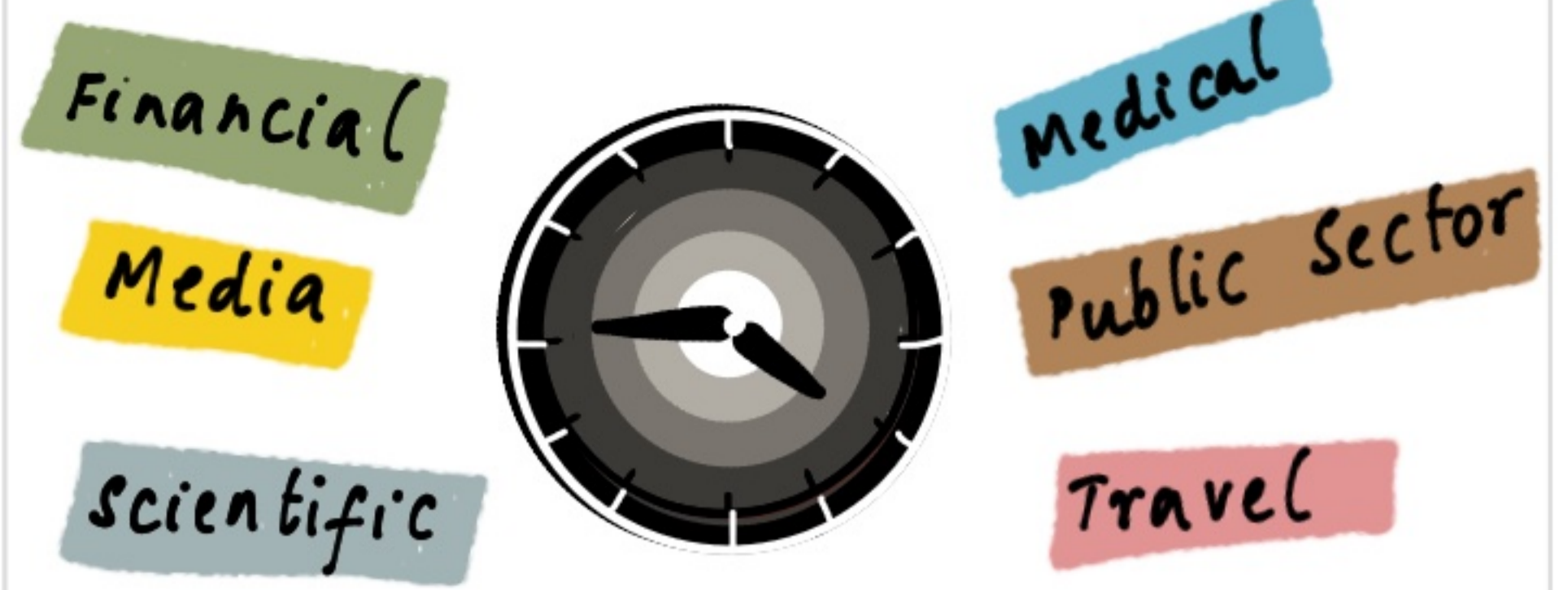
IMAGINE A
TIMELINE FOR
IMPLEMENTING P&C

JANUARY	2030
FEBRUARY	
MARCH	2037
APRIL	
MAY	2040
JUNE	2043
JULY	

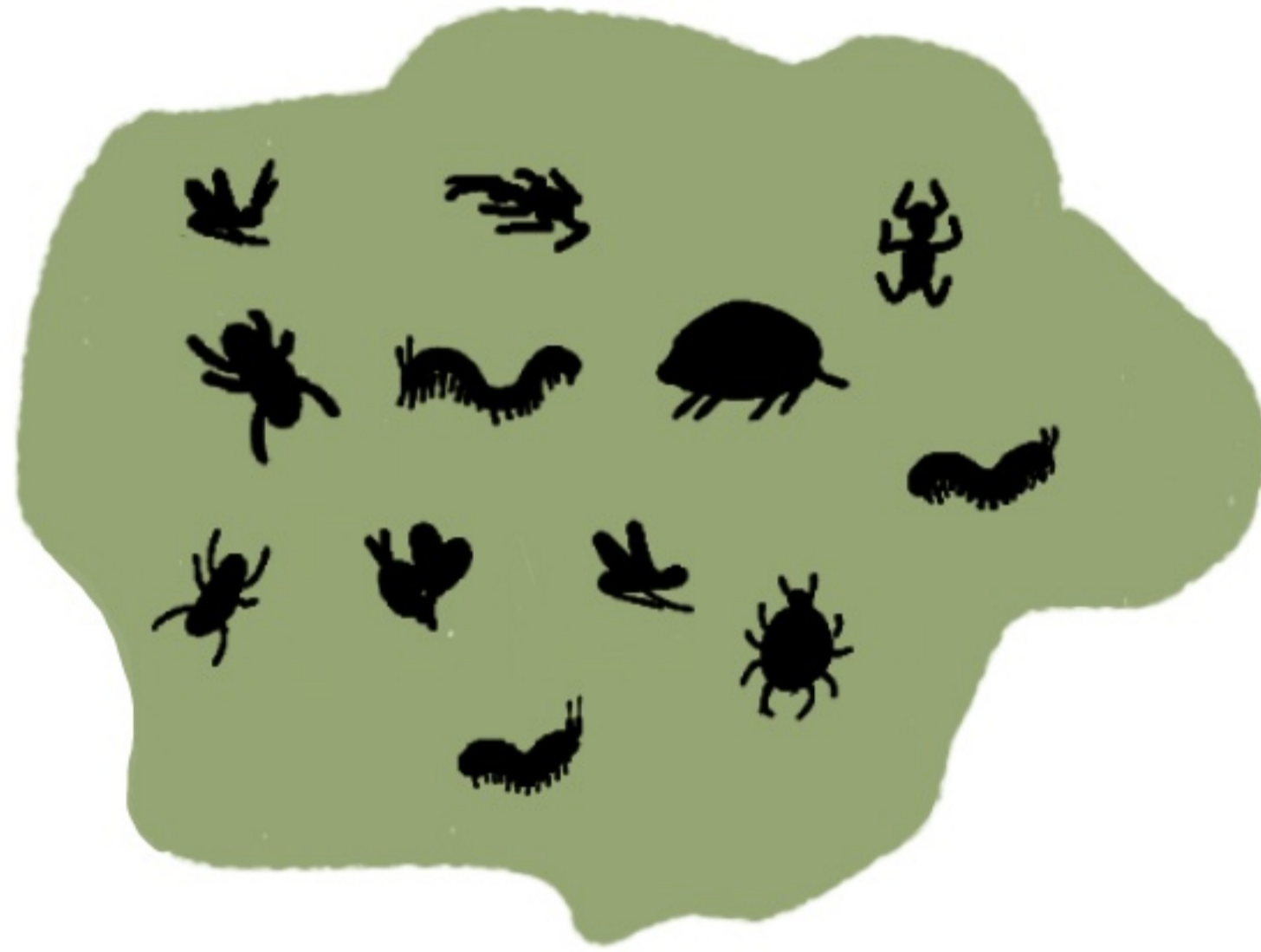
TO IDENTIFY RISK AREAS



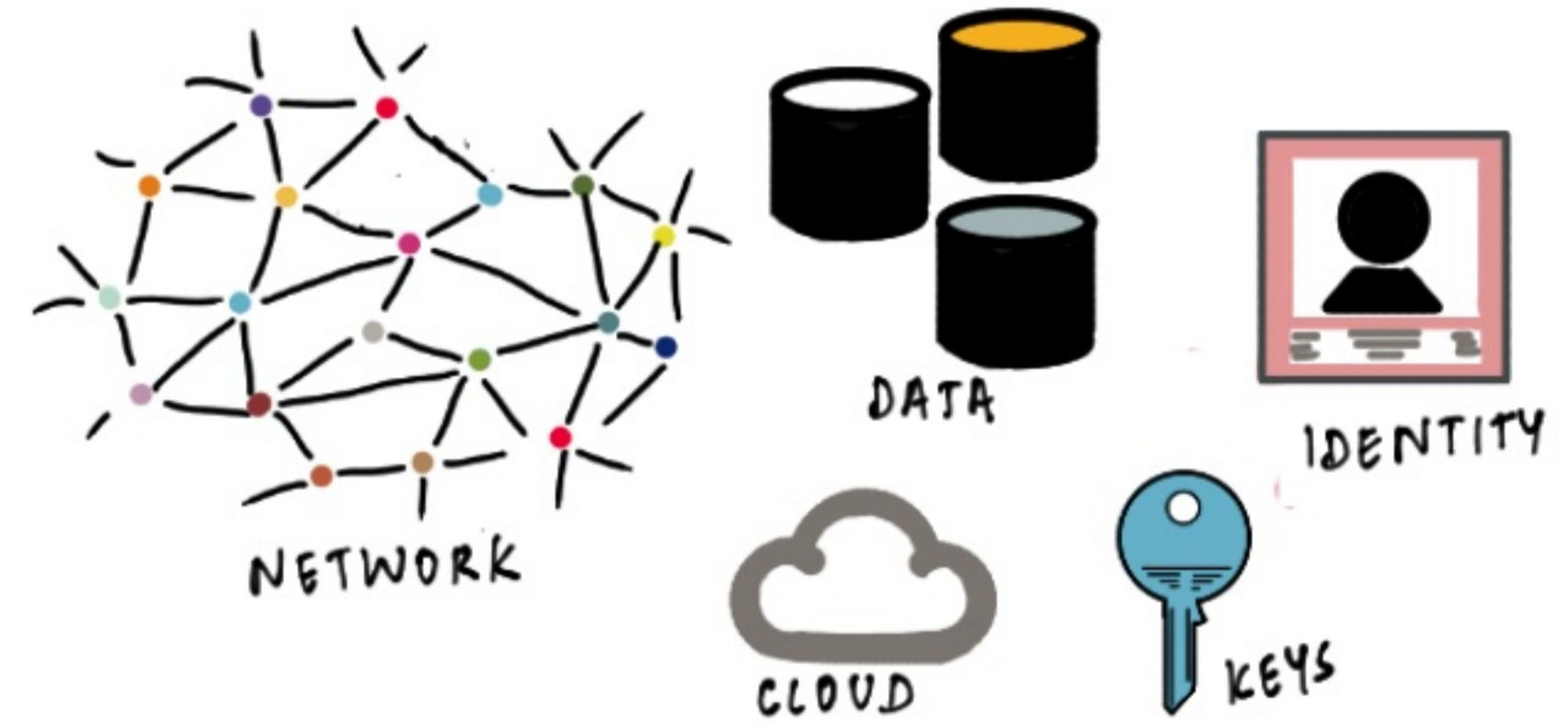
IS THERE ANY DATA THAT IS SENSITIVE OR CONFIDENTIAL?



HOW LONG WILL THE DATA NEED TO BE KEPT SAFE?



IS THE DATA PUBLIC FACING?



DOES THE INFRASTRUCTURE NEED UPDATE?

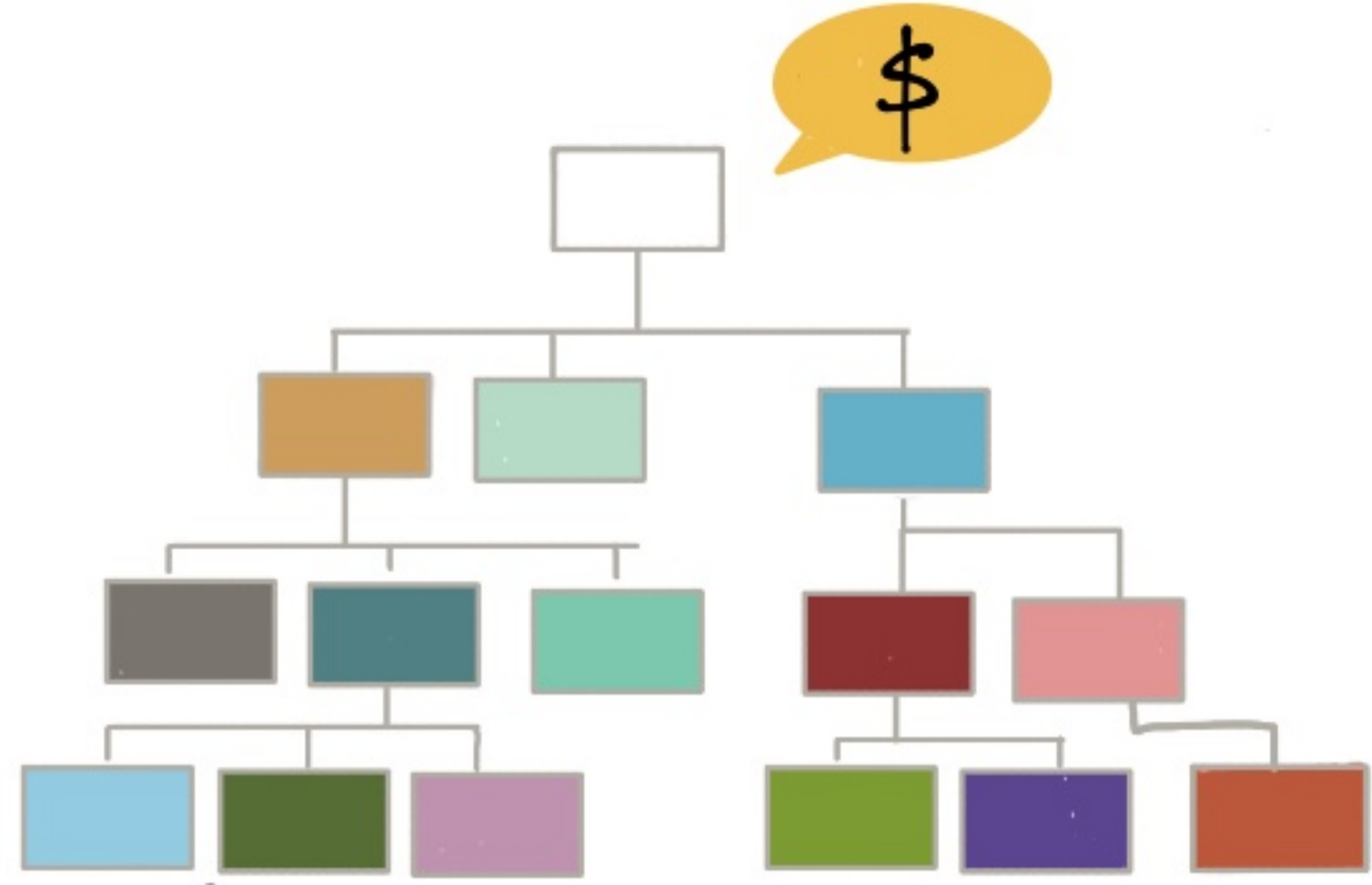


WHICH COMMUNICATIONS SYSTEMS NEED TO BE SECURED?

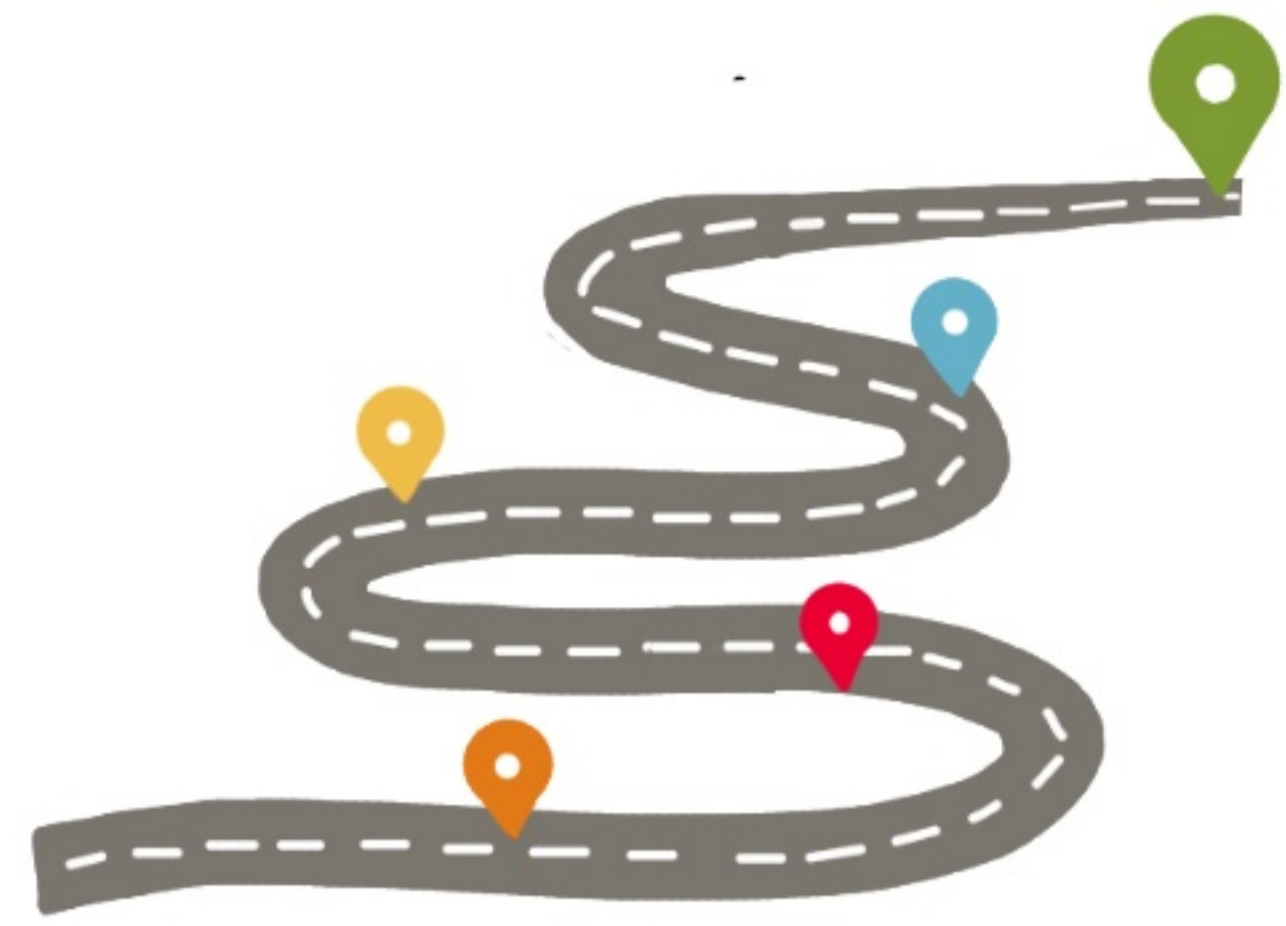


WHICH APPS ARE AFFECTED? (INTERNAL AND EXTERNAL)

OTHER CONSIDERATIONS



PEOPLE / TEAMS WHO
NEED TO BE MADE AWARE



VENDOR ROADMAPS

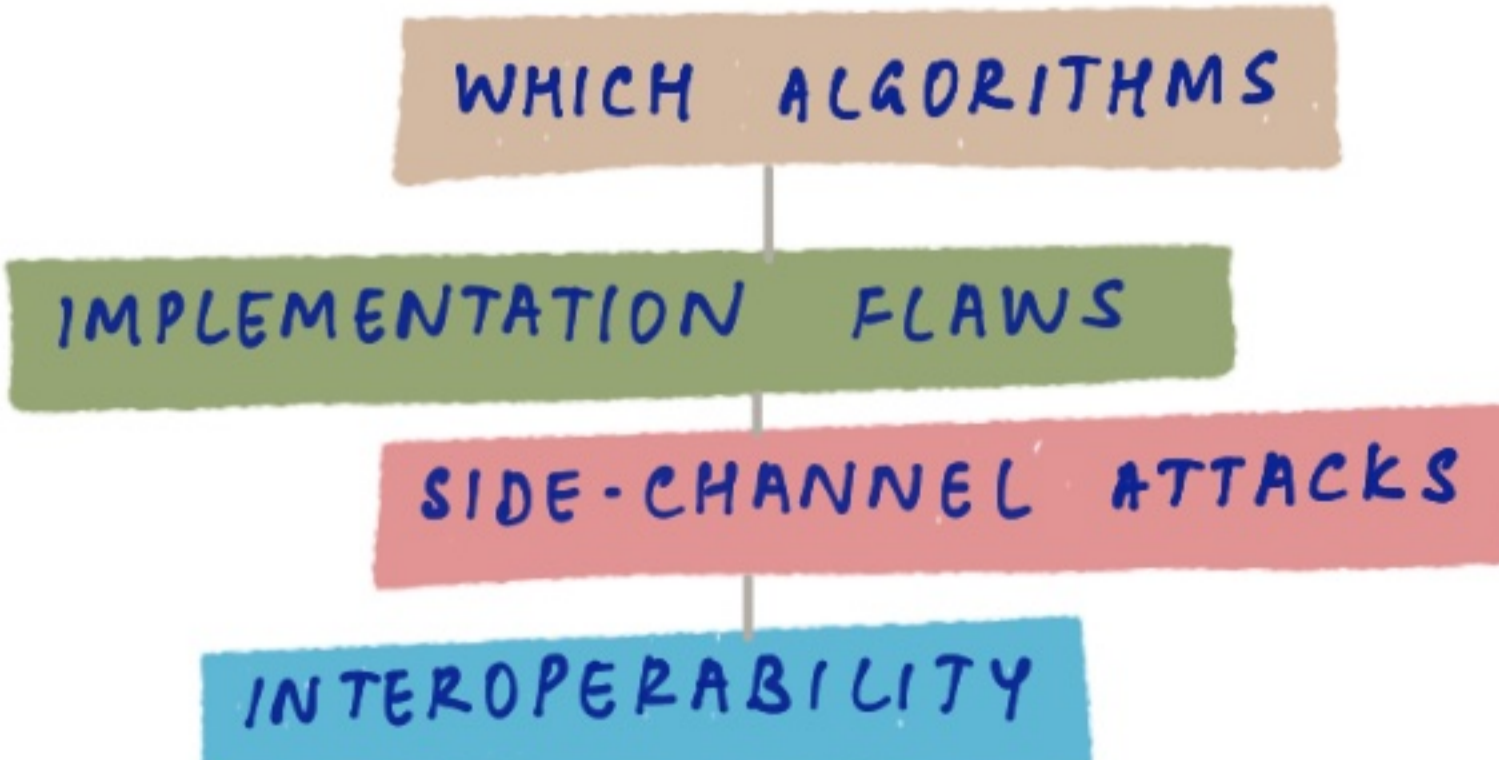
EASY TO REPLACE?

1. M N	✓
2. M N	✓
3. M N	✗
4. M N	✓
5. M-N-M.	?

CRYPTOAGILITY



TRANSITION STRATEGY



EXPERIMENT, TEST &
IMPLEMENT

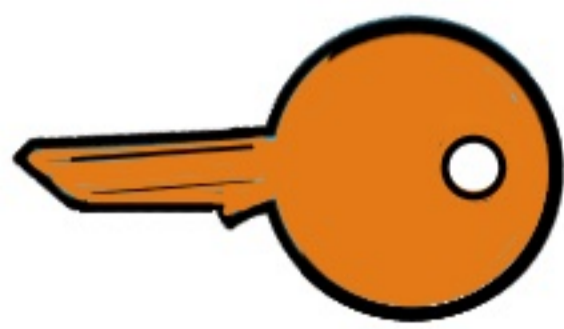
GUIDANCE FROM
REGULATORY BODIES :

NIST ANSSI
BSI IEEE

STAY INFORMED

CHALLENGES IN PQC

PQC INVOLVES TRADEOFFS



KEY SIZES

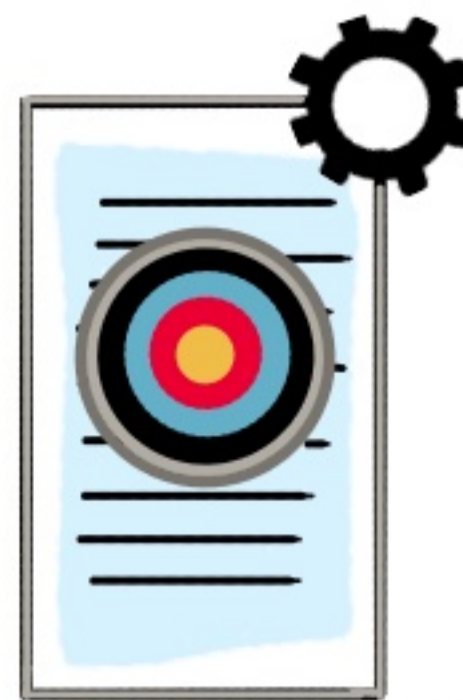


EFFICIENCY



POWER CONSUMED

UNDISCOVERED VULNERABILITIES



Quantum-safe

SIDE CHANNEL
ATTACKS
POSSIBLE

NOT FULLY
TIME-TESTED

COST OF TRANSITION

LIKELY TO BE HIGH



Hardware

Software

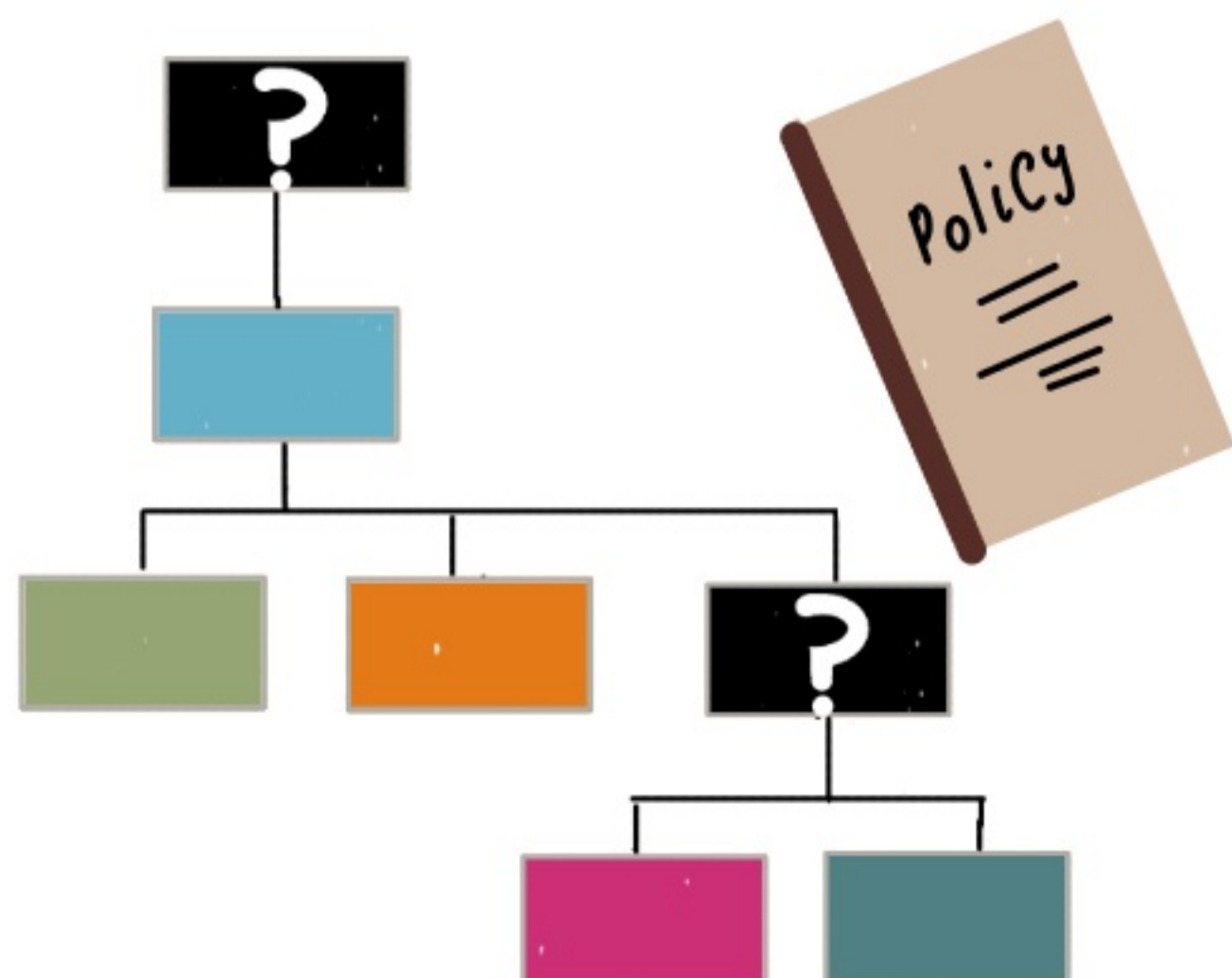
Maintenance

FEELS NON-URGENT



QUANTUM COMPUTERS
ETA: UNKNOWN

UNCLEAR GOVERNANCE



LEGAL ISSUES

NEED CLEAR UP-TO-DATE

- PRIVACY LEGISLATIONS
- REGULATIONS FOR DIGITAL SIGNATURES
- & MORE

NOT IN SCOPE

SPECIFIC NUMBERS ASSOCIATED TO THE KEY SIZES OR COMPUTATION TIME REQUIRED - AS IT WILL DEPEND ON FINAL IMPLEMENTATION

STEPS FOR NAVIGATING A HYBRID APPROACH TO PQC

EXPLORING OTHER BASES FOR POST QUANTUM CRYPTOGRAPHY SUCH AS CELLULAR AUTOMATA OR DIOPHANTINE EQUATIONS

HOW AI AND QUANTUM MIGHT WORK TOGETHER

WE SHOULD NOT GET TOO COMFORTABLE
WITH THE TERM 'SECURE'

- VINT CERF

SEPTEMBER 2023
PRC PANEL DISCUSSION

MY REFERENCES

Fundamental concepts

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- What is it going to take to break cryptography with a quantum computer by **PKI Consortium**
- NIST Pages - csrc.nist.gov/projects/post-quantum-cryptography
- The new millennium bug: everything you need to know about Y2K : weforum.org
- Panel Discussion: Post-Quantum Cryptography | September 23 youtube.com by Heidelberg Laureate Forum
- Status Update from NIST on youtube.com by PKI consortium

Algorithms : Videos on Youtube.com

- Quantum Algorithms and Post-Quantum Cryptography by **Simons Institute**
- ISBA2022: Workshop | Demystifying Quantum Part 2 by **Parallel Chain Lab**
- Lattices and Kyber PQC Presentation by **Mojtaba Bisheh Niasar**
- Kyber and Post Quantum Crypto How does it work by **All Hacking Cons**
- Learning with errors: Encrypting with unsolvable equations by **Chalk Talk**
- Lattices: Algorithms, Complexity, and Cryptography by **Simons Institute**
- Jintai Ding - State of Art of MPKC by **PQCrypto 2016**
- Code-based Cryptography by **PKI Consortium**
- Code based Crypto by **USF Crypto Center**
- Oil and Vinegar variables : **Bill Buchanan**

Implementation

- Paper - Challenges in the Transition towards a Quantum-safe Government : tudelft.nl
- Transitioning Organizations To Post Quantum Cryptography youtube.com by SandboxAQ